Recommender systems

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Introduction

Chapter 1

1.1 Recommendation systems definition, application domain

🕮 1.1.1

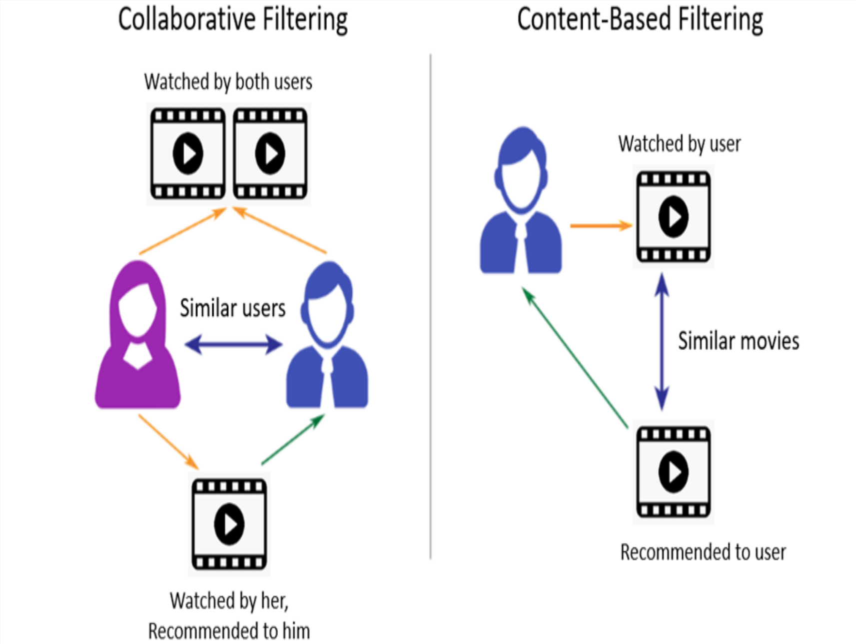
Recommender systems (or also Recommendation systems) are special software applications using artificial intelligence. Typical definition is given by [Wikipedia], cite: „*Recommender system is a subclass of information filtering system that provide suggestions for items that are most pertinent to a particular user. Typically, the suggestions refer to various decision-making processes, such as what product to purchase, what music to listen to, or what online news to read. Recommender systems are particularly useful when an individual needs to choose an item from a potentially overwhelming number of items that a service may offer.*“

Recommender systems use a number of different technologies. We can classify these systems into two broad groups, according to [Mining of Massive Datasets. Chapter 9 Recommendation systems]:

• *Content-based filtering systems* examine properties of the items recommended. For instance, if a Netflix user has watched many cowboy movies, then recommend a movie classified in the database as having the “cowboy” genre.

• *Collaborative filtering systems* recommend items based on similarity measures between users and/or items. The items recommended to a user are those preferred by similar users. This sort of recommendation system can use the groundwork on similarity search on clustering. However, these technologies by themselves are not sufficient, and there are some new algorithms that have proven effective for recommender systems.

The main idea of both systems is present in the picture:



Content-based filtering works on the principle of similar content. If the user is watching a movie, the system will check other movies of similar content or the same genre as the movie the user is watching. Unlike collaborative filtering, content-based approaches will use additional information about the user and/or items to make predictions. They use descriptive keywords associated with each item to make recommendations. This is quite useful because the only ranking history we need to make predictions is the history of the target user. We can extract features from the movie description and compare what the user prefers.

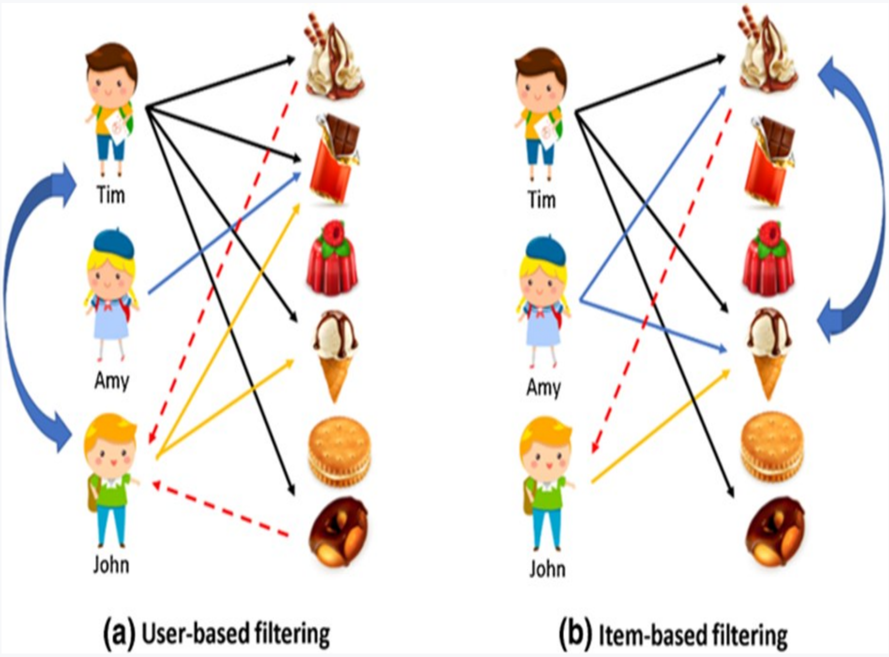
The collaborative filtering systems are the most desired, widespread and advanced technology available in the market. A key concept in collaborative filtering methods is that they are collaborative, i.e. they use the evaluations of other users. If you're trying to guess whether you'll like a certain movie, you can ask people with similar tastes what they think about the movie. You can also ask these people what other movies they liked and get a list of recommendations. It collects object ratings or recommendations, recognizes common characteristics of users based on their ratings, and generates new recommendations based on - user comparisons. Collaborative filtering can be used in two ways:

1. *Memory-based* or *neighborhood-based* methods are a simple way to use other users' ratings to predict another ratings. They are used in two variants: custom and item-based.

2. *Model-based* methods take collaborative filtering as a step further and use machine learning and probabilistic models such as decision trees, latent factor models, and neural networks as classification methods.

Memory-based or **neighborhood-based** system collects behavioral information about how you interact with items - what ratings you have, how you rate, what you view and/or what you purchase. One typical approach is to create a user rating matrix containing user ratings on movies. We find similarities and make recommendations. Similarity is not only limited to the user's taste, in addition, similarity between different items can also be considered. If we have a large amount of information about users and items, the system will provide more effective recommendations.

Example:



In the collaborative filtering above, there are three users Tim, Amy and John who are interested in desserts. The system detects users who have the same taste in purchasing products, and the similarity between users is calculated based on purchasing behavior.

User-based collaborative filtering: Tim and John are similar because they bought similar products.

Item-based collaborative filtering: the system checks for items similar to items the user has purchased. The similarity between different items is calculated based on items and not users for prediction. Tim and Amy bought an ice cream sundae and an ice cream cone, so it turns out they have similar tastes.

The application domain for recommender systems is very wide. Typical application are known form movie databases, which we will also use. But they are many applications for business purposes [Recommender Systems For Business - A

Gentle Introduction] available. Many possibilities for recommender systems are still waiting for solution.

1.1.2

The main methods used in recommender systems are:

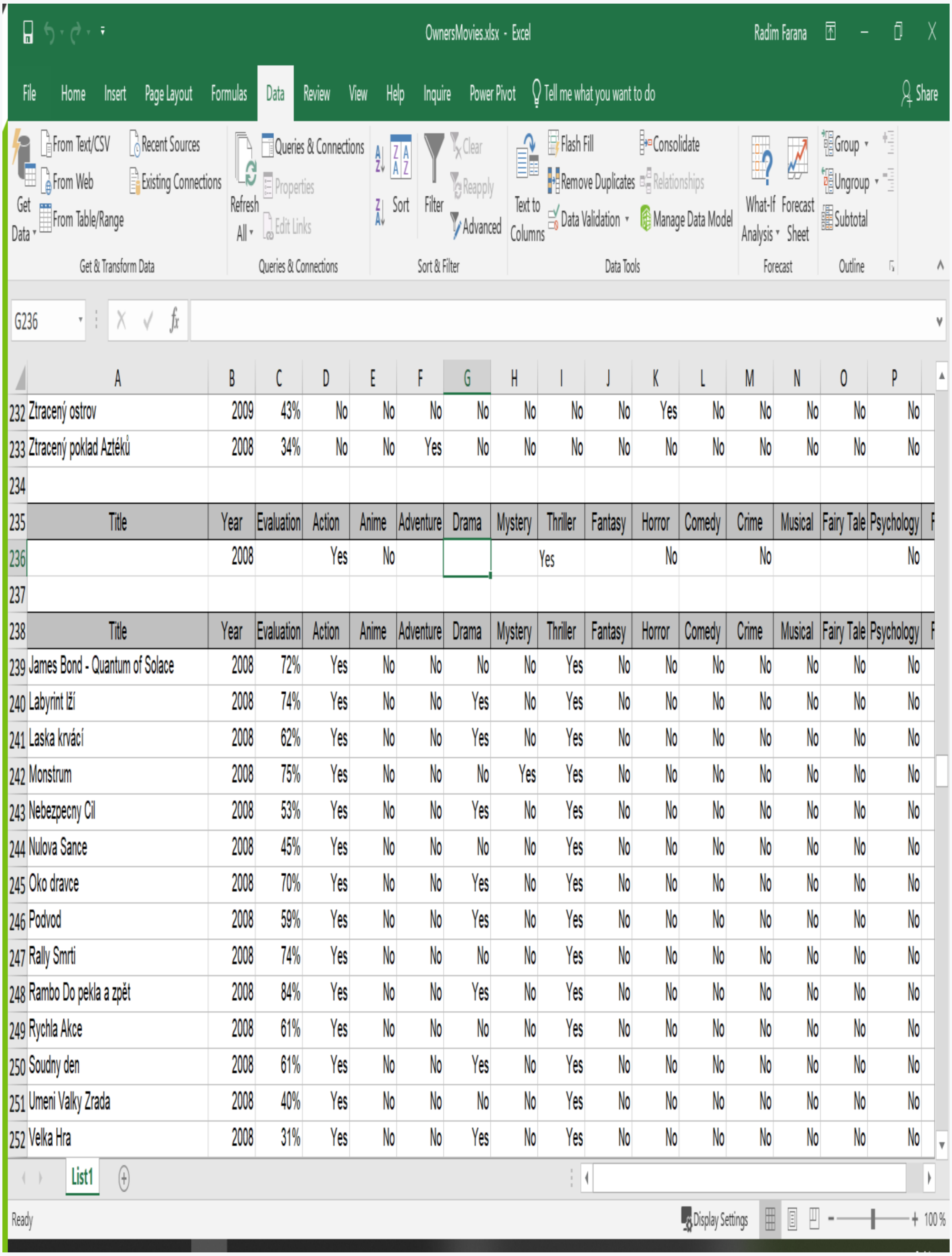
- Memory-based methods
- Model-based methods
- Matrix-based methods
- Mountain-climbing methods

1.2 Easy example – movie selection based on parameters in MS-Excel

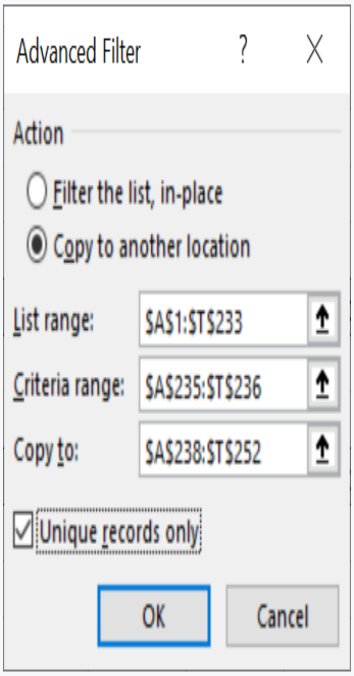
1.2.1

Typical recommender systems applications are known from the movie industry. Movie portals offers so many movies that it is difficult to navigate them and it is very difficult to choose a suitable movie for a specific occasion. To facilitate the choice, we can define a set of parameters (for example “Drama”, “Sci-fi”, etc.) and assign them to all movies.

The selection problem could be presented by the use of some owner’s movie database in MS-Excel (that is why the names of movies are in owner’s language, file is for your experiments available here: https://priscilla.fitped.eu/data/recommender_systems/OwnersMovies.xlsx), which allow us to select appropriate movies with the help of more parameters:



For movie section based on selected parameters we can use the standard tool "Advanced filter":



Even this kombination of parameters (Action Thriller which is not Anime, Horror, Crime nor Psychology) gave us 14 suggested results.

1.2.2

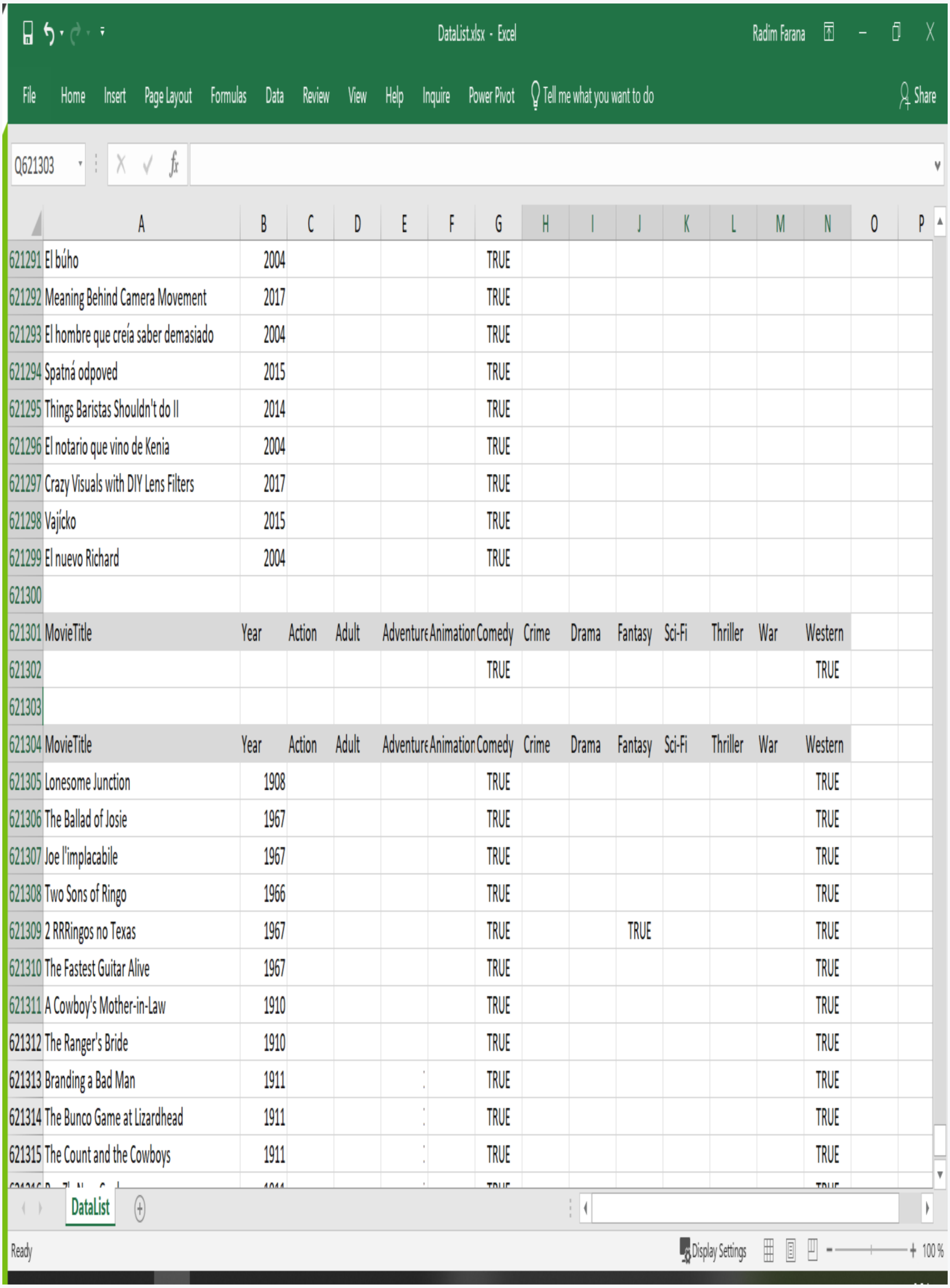
Which kind of filtering are we used in the example of movie selection, based on parameters in the file https://priscilla.fitped.eu/data/recommender_systems/OwnersMovies.xlsx?

- Content-based filtering
- Collaborative filtering

1.2.3

Even in this short list of movies we must use more parameters to obtain short list of possible movies. To understand how complicate problem is to recommend appropriate list of movies you can use for non-commercial purposes movie database available on web: https://datasets.imdbws.com/, documentation for these data files can be found on http://www.imdb.com/interfaces/ (encoding Unicode (UTF-8)).

Having so long list of movies it is practically impossible to obtain some short list of movies (the original file for your experiments is available here: https://priscilla.fitped.eu/data/recommender_systems/DataList.xlsx), for example:



	A	B	C	D	E	F	G	H	I	J	K	L	M	N
621291	El búho	2004					TRUE							
621292	Meaning Behind Camera Movement	2017					TRUE							
621293	El hombre que creía saber demasiado	2004					TRUE							
621294	Spatná odpoved	2015					TRUE							
621295	Things Baristas Shouldn't do II	2014					TRUE							
621296	El notario que vino de Kenia	2004					TRUE							
621297	Crazy Visuals with DIY Lens Filters	2017					TRUE							
621298	Vajícko	2015					TRUE							
621299	El nuevo Richard	2004					TRUE							
621300														
621301	MovieTitle	Year	Action	Adult	Adventure	Animation	Comedy	Crime	Drama	Fantasy	Sci-Fi	Thriller	War	Western
621302							TRUE							TRUE
621303														
621304	MovieTitle	Year	Action	Adult	Adventure	Animation	Comedy	Crime	Drama	Fantasy	Sci-Fi	Thriller	War	Western
621305	Lonesome Junction	1908					TRUE							TRUE
621306	The Ballad of Josie	1967					TRUE							TRUE
621307	Joe l'implacabile	1967					TRUE							TRUE
621308	Two Sons of Ringo	1966					TRUE							TRUE
621309	2 RRRingos no Texas	1967					TRUE			TRUE				TRUE
621310	The Fastest Guitar Alive	1967					TRUE							TRUE
621311	A Cowboy's Mother-in-Law	1910					TRUE							TRUE
621312	The Ranger's Bride	1910					TRUE							TRUE
621313	Branding a Bad Man	1911					TRUE							TRUE
621314	The Bunco Game at Lizardhead	1911					TRUE							TRUE
621315	The Count and the Cowboys	1911					TRUE							TRUE

1.2.4

When using the list of available movies: https://priscilla.fitped.eu/data/recommender_systems/DataList.xlsx, how complicated is it to obtain a short list of recommended movies, based on the parameters selection?

- It is a piece of cake.
- It is practically impossible

Fuzzy logic systems

Chapter 2

2.1 Fuzzy logic principles

2.1.1

Human decision is mostly based on vague information like "add a pinch of salt", "it's quite hot", "not far", etc. **Fuzzy logic** fills this expression with mathematical content, which is why even a computer can understand it. In this way, processing imprecise (indeterminate) input data is also possible - which is not possible with classical Boolean algebra.

Fuzzy logic has been invented by Prof. Zadeh [Zadeh 1965] and used to describe uncertain systems [Zadeh & Kacprzyk 1992] since the ’60s of the 20th century.

Fuzzy logic consists in extending logical operators to fuzzy sets. The fuzzy set theory consists in the introduction of the so-called degree or membership of an element's belonging to a set, which can take on values from the interval <0, 1>, in contrast to classical set theory, where each element either belongs to the set or does not. The degree of belonging of an element (e.g. instantaneous temperature) to a fuzzy set (temperature) can take on any value from zero to one (inclusive). With a value of 0, the element definitely does not belong to the set, 0.2 means hardly, 0.5 maybe, 0.8 almost certainly, and 1 means definitely belonging to the set. This elegantly avoids the situation where, in classical logic, we label a temperature of 25°C as pleasant and 24.9°C as unpleasant.

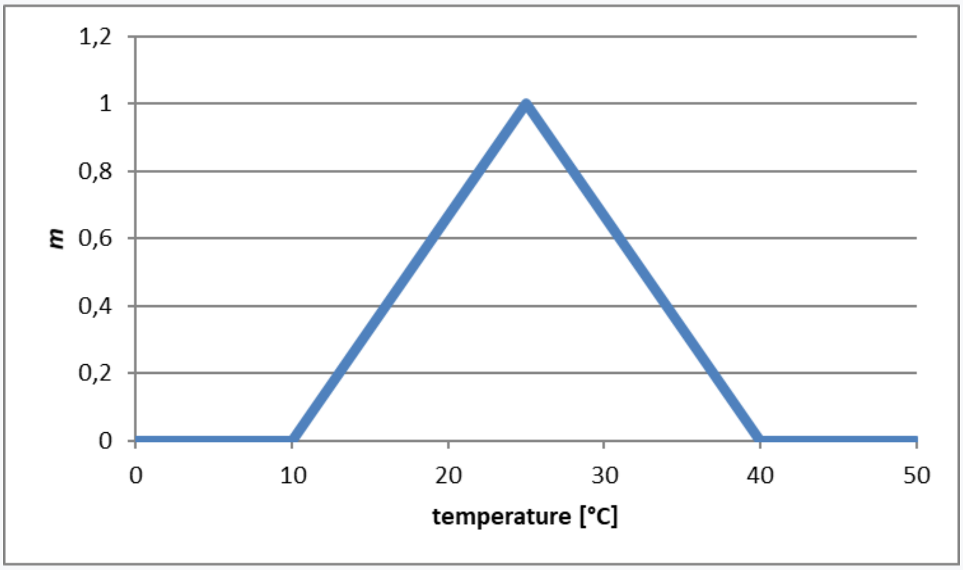
Example:

Let's have a set of air temperatures labeled *Pleasant Temperature*. Intuitively, we feel that a temperature of 0°C is definitely not pleasant, 10°C is hardly a pleasant temperature, 20°C is almost certainly pleasant, and 25°C is definitely a pleasant temperature, on the other hand, 35°C will also hardly be pleasant. So we can establish a table of the degree of membership of the actual temperature to the set "*Pleasant temperature*":

temperature	degree of membership *m*	linguistic description
5 °C	0	it's definitely not pleasant
10 °C	0,3	maybe barely
15 °C	0,5	perhaps
20 °C	0,8	almost certainly
25 °C	1,0	most certainly
35 °C	0,5	barely

Obviously, different people may have different opinions about verbal expression and degree of membership.

Example of membership function:

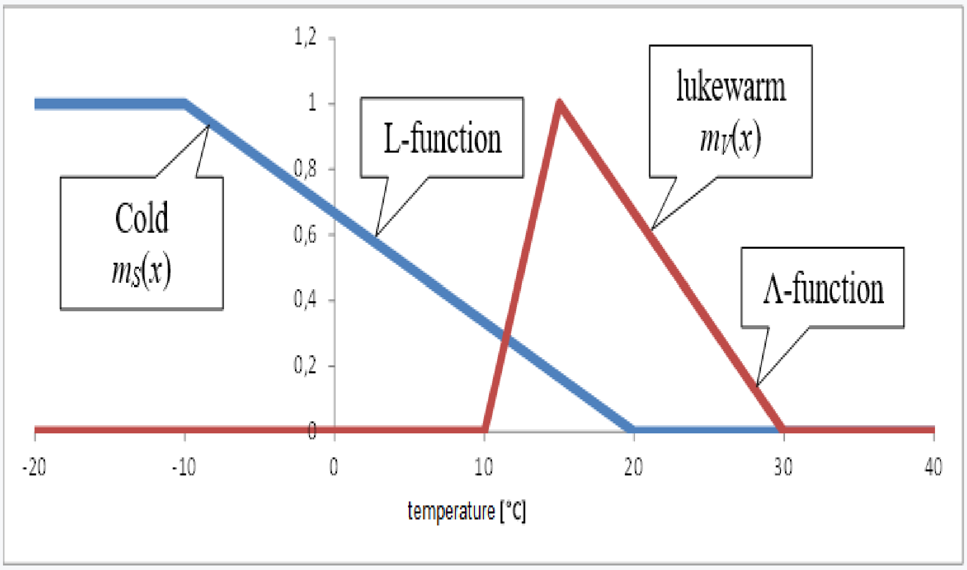


At the same time, the degree of membership is not related to the probability of the phenomenon (it also acquires the values 0 ÷ 1), because it does not tell us whether the phenomenon will occur. It only determines with what "strength" a particular value belongs to the selected set. If the function characterizes the degree to which an element belongs to a set, then we refer to these sets as **fuzzy sets**. If we want to use the empirical experience of operators, personnel, and experts, we cannot do without the introduction and use of language variables.

A **linguistic variable** is a variable whose values are expressions of a language. We can interpret the value of a language variable as a fuzzy set. A set of values is referred to as a set of *n* linguistic terms. The meaning of terms is defined by the universe, which we understand as a universal set. E.g. when regulating the temperature of the bath, we can understand the temperature of the water as a language variable named "*Bath temperature*". We measure temperature in centigrades (degrees of Celsius). However, the quantitative expression of the temperature of the bath in the colloquial language does not have to be expressed only in degrees, but in commonly used expressions such as: the bath is ICE COLD, COLD, MOIST, WARM, etc. We can then designate an element from the set of temperatures as the value of the language variable

"*Bath temperature*": {icy(L), cold(S), lukewarm(V), hot(H)}.

The linguistic quantification of temperatures introduced in this way using natural language expressions (e.g. cold) represents terms whose meaning is vague and is modeled using fuzzy sets and is defined by the characteristic function $m_S(x)$. The characteristic function $m_S(x)$ is called the membership function $m_S(x)$ for fuzzy sets. It characterizes the degree to which a given element belongs to a given set, from a value of 0, when the element definitely does not belong to the set, to a value of 1, when the element definitely belongs to the set. As an example of membership functions, we present their cold and lukewarm membership functions $m_S(x)$ and $m_V(x)$ for fuzzy sets. The meaning of the terms *cold* and *lukewarm* is modeled by the membership function on a certain temperature interval (universe) in centigrades.



We will explain the membership function of a fuzzy set using the following example:

If we measure the temperature *x* = 20°C, then $m_S(x)$ = 0, and certainly this measured temperature does not belong to the term - language value cold.

If we measure the temperature x = 0°C, then $m_S(x)$ = 0.7, which indicates that this measured temperature belongs to the term - language value cold with the degree of affiliation 0.7.

If we measure the temperature x = -10°C, then $m_S(x)$ = 1 and it is clear that this measured temperature belongs to the term - language value cold with the degree of affiliation 1.

If we measure the temperature x = -20°C, then $m_S(x)$ = 1 and this measured temperature also belongs to the term - language value cold with the degree of affiliation 1.

If we measure the temperature x = +15 °C, then $m_S(x)$ = 0.25, and this measured temperature belongs to the term - linguistic value cold with the degree of affiliation 0.25. But be careful, the membership function $m_V(x)$= 1, from which it follows that this measured temperature also belongs to the set - term lukewarm with membership degree 1.

The process of assigning measured values of input quantities to fuzzy sets using membership functions is referred to as **fuzzification**.

Standard membership functions are used for many purposes: Λ-function (triangular function), L-function (see previous figure), Π-function (trapezoidal function) see Γ-function, S-function, and Z-function.

2.1.2

Fuzzy logic has been invented by:

- Lofti Zadeh
- Radim Farana
- Cyril Klimeš
- Constantin, the philosopher

2.1.3

Basic operations with fuzzy sets

Fuzzy sets can be considered a generalization of classical crisp sets. A classical set can be considered a special case of a fuzzy set, whose membership function takes only the values 0 and 1.

On fuzzy sets, we can use operations similar to those we used on classical sets. For our purposes, we will only list the three basic operations (complement, intersection, and union).

Complement of fuzzy set, the complement of set A, C = NOT A:

$$m_C(x) = 1 - m_A(x)$$

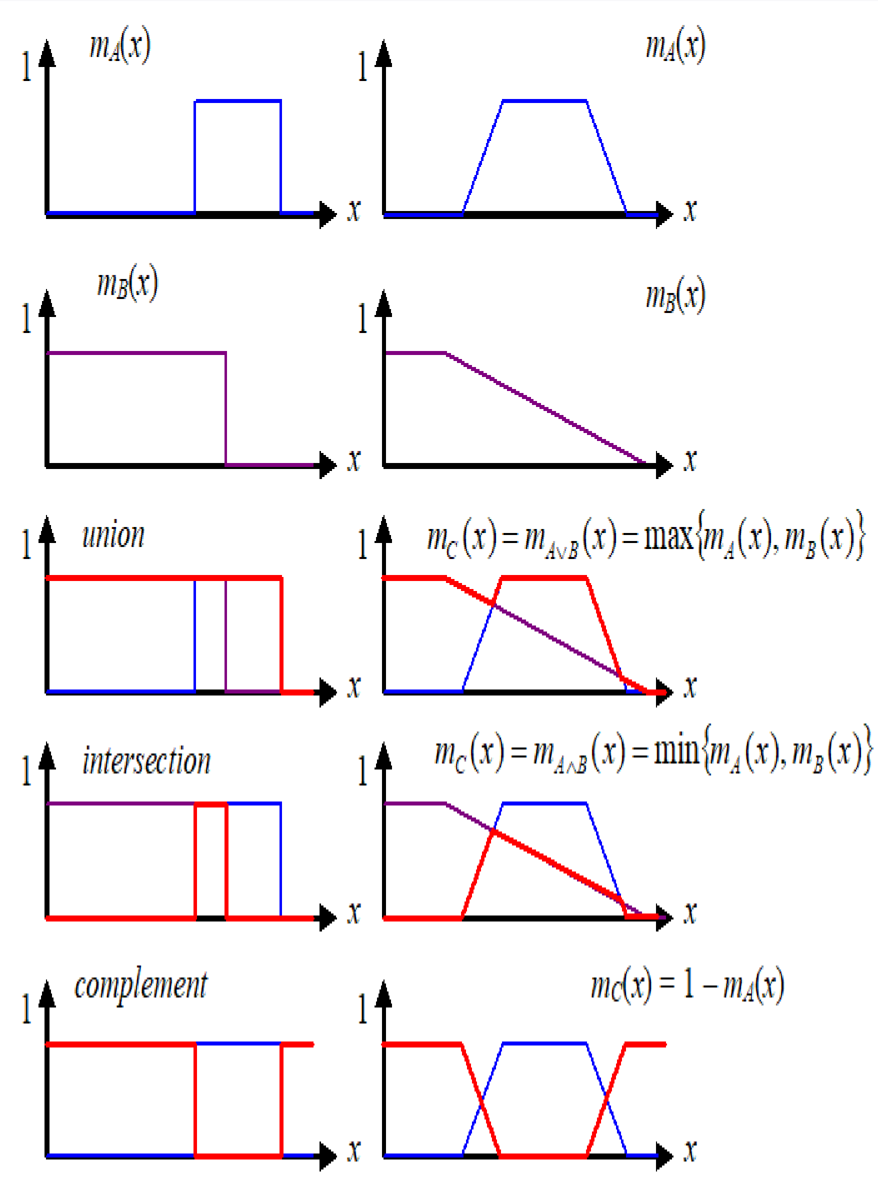
Intersection of fuzzy sets (logical product) C = A AND B

$$m_C(x) = m_{A \wedge B}(x) = \min\{m_A(x), m_B(x)\}$$

Union of fuzzy sets (logical sum) C = A OR B

$$m_C(x) = m_{A \vee B}(x) = \max\{m_A(x), m_B(x)\}$$

These operations are shown in the figure:



2.1.4

We know three basic operations with fuzzy sets, they are:

- Complement, intersection, and union
- Compartment, international, and unia
- Concatenation, interruption, and unique

2.1.5

Fuzzy rules

In general, the logical inference is based on the evaluation of decision rules in the form of conditional statements

IF .... THEN.....

For inferring knowledge, the condition is expressed in the form of an implication of two fuzzy statements mostly as:

IF <fuzzy expression> THEN <fuzzy expression>

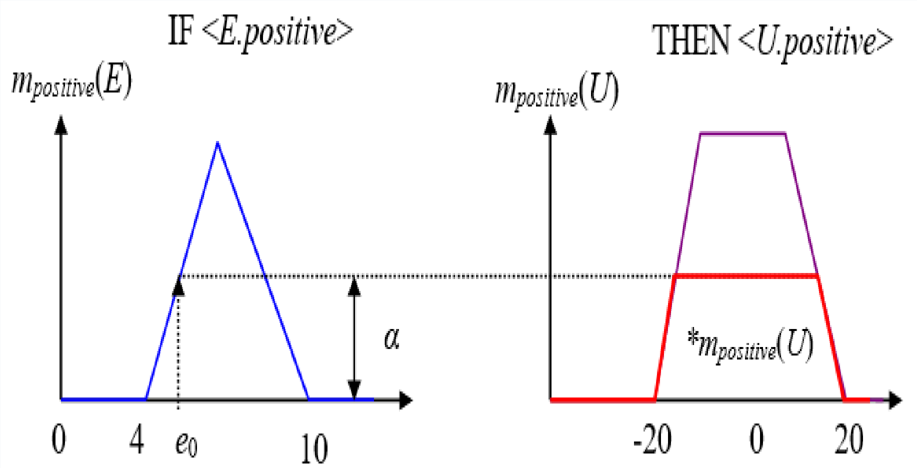
This condition is referred to as the "*fuzzy IF-THEN rule*". The first fuzzy expression set, which is often a compound statement, is called the **antecedent**, where individual parts of the statement are bound by logical conjunctions. The second fuzzy statement is **consequential**.

Example:

A simple fuzzy rule:

IF <E.positive> THEN <U.positive>

In the decision rule, the antecedent contains a linguistic variable E (regulatory deviation), whose value is positive and has the membership function $m_{value}(E)$. The consequent contains a language variable U (action quantity) with a positive value, whose membership function is $m_{positive}(U)$,



If we measure the crisp value of the regulation deviation e_0, then we can use the membership function $m_{positive}(E)$ to subtract the degree of membership α with which the measured value belongs to the set of values *E*.positive. However, our task is to find the corresponding fuzzy set of the consequent for the measured crisp value. The most common procedure for determining this set is based on the logical assumption that the consequence - the consequent can have at most the same degree of membership as the condition - the antecedent. The degree of membership of the measured "crisp" value e_0 therefore determines the level that cuts the output fuzzy set U of the consequent. The membership function of the consequent is then $*m_{\text{kladná}}(U)$ (outline in bold).

A generalization of this principle to a two-dimensional antecedent

IF (X is positive small) AND (Y is positive medium) THEN (U is negative medium)

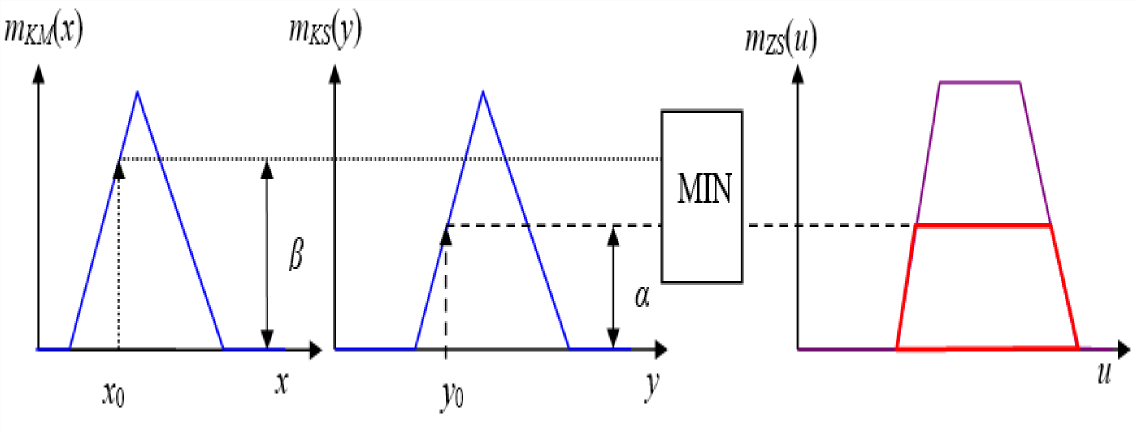
There are several ways to model the meaning of the fuzzy IF-THEN rule, but from the point of view of many applications, the most important is the following method (by Mamdani [Mamdani & Assilian 1975]), which defines the membership function of the consequent as:

$$m_{IM} = \min\{m_A(x_1), m_B(x_2)\}$$

Minimization expresses the fact that the consequence (consequent) can have at most the same degree of membership as the condition (antecedent). Finding the output set for a single rule and a two-dimensional dependency differs depending on whether the AND or OR operator is used. If the operator is AND, then we select the minimum of the corresponding values of the membership functions, see the next figure.

IF <x.KM> AND <y.KS> THEN <u.ZS>

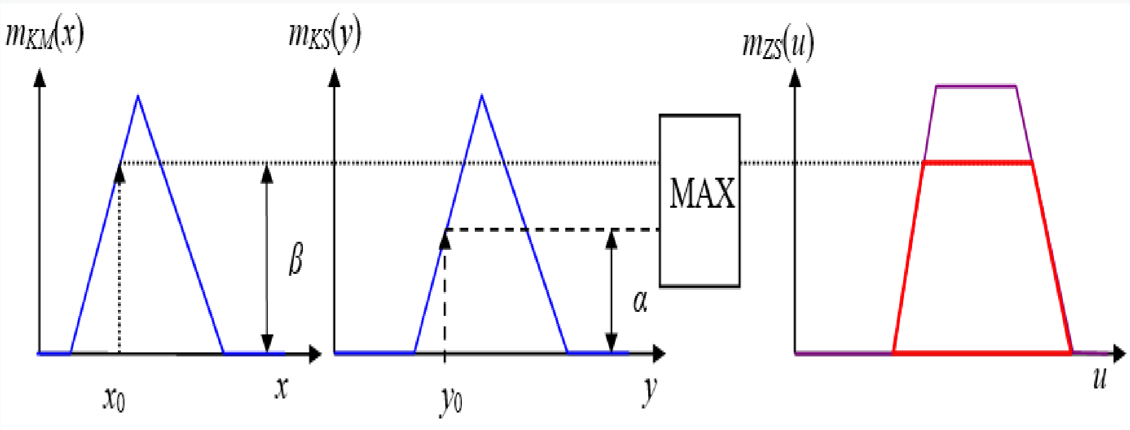
Trimming the membership function of the consequent at level α, which corresponds to the minimum of the two membership functions of the input values.



If the operator is OR, then we select the maximum of the corresponding membership functions, see the next figure.

IF <x.KM> OR <y.KS> THEN <u.ZS>

Trimming the membership function of the consequent at the β level that corresponds to the maximum of the two membership functions of the input values.



Finding the output set for two rules and two-dimensional dependence and Mamdani's method is shown in the next figure.

IF <x.KM> AND <y.KM> THEN <u.KM> ELSE IF <x.KS> AND <y.KS> THEN <u.KS>

For two fuzzy IF-THEN rules, their meaning is modeled by these membership functions.

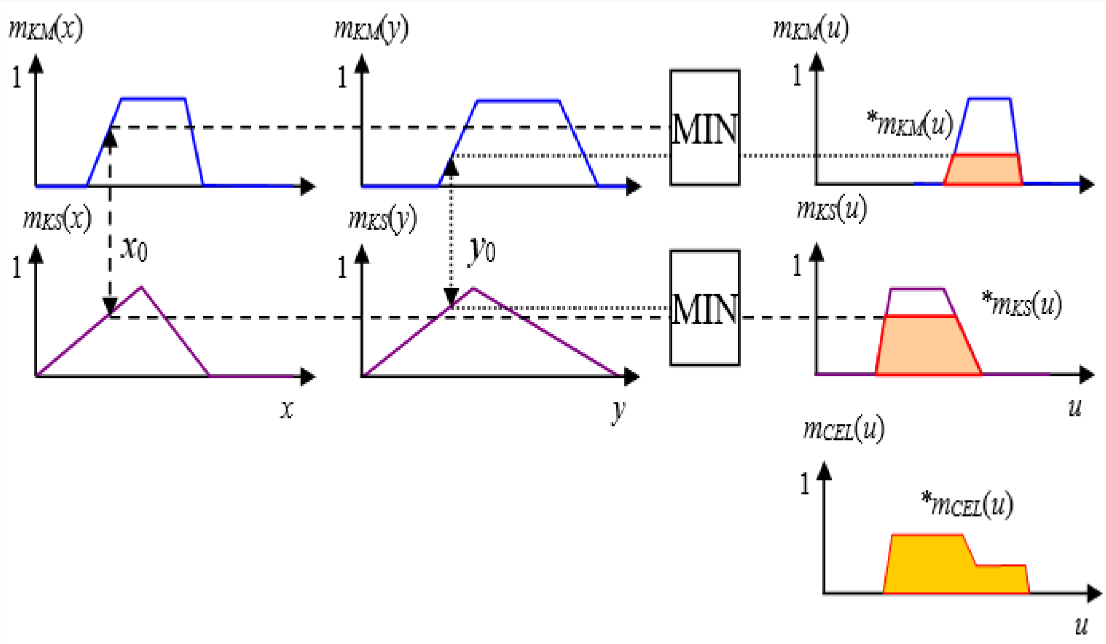
$$\alpha_1 = m_{KM}(x) \wedge m_{KM}(y) = \min\{m_{KM}(x), m_{KM}(y)\}$$

$$\alpha_2 = m_{KS}(x) \wedge m_{KS}(y) = \min\{m_{KS}(x), m_{KS}(y)\}$$

For the consequents of both implications, we get:

$$*m_{KM}(u) = \alpha_1 \wedge m_{KM}(u) = \min\{\alpha_1, m_{KM}(u)\}$$

$$*m_{KS}(u) = \alpha_2 \wedge m_{KS}(u) = \min\{\alpha_2, m_{KS}(u)\}$$



The consequents of both implications $*m_{KS}(u)$ and $*m_{KM}(u)$ determine their partial shares in the size of the action quantity. Intuitively, it is possible to interpret the effect of both partial output terms as their logical sum. Then for the output fuzzy set of both effects, we get:

$$*m_{CEL}(u) = \max\{\min\{\alpha_1, m_{KM}(u)\}, \min\{\alpha_1, m_{KM}(u)\}\}$$

This approach can be extended to any number of decision rules.

2.1.6

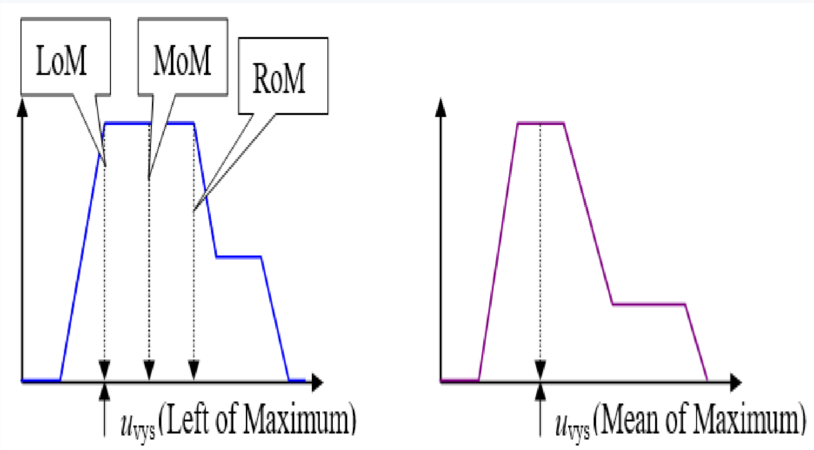
For inferring knowledge, the condition is expressed in the form of an implication of two fuzzy statements mostly called:

- Fuzzy IF-THEN rules
- Fuzzy FOR-THAN rules
- Fuzzy BETWEEN-AND rules
- Fuzzy WHILE-END rules

📖 2.1.7

Defuzzification

The result of the action of the block of decision rules is a set of membership functions for individual terms of the output language variables. The membership function of the output set is given by the union of trimmed membership functions (Mamdani's method). After the practical implementation of action interventions, it is necessary to assign the output language variables a crisp value of the action quantity within the permissible range. This process of "approximating fuzzy terms" by a crisp valued action quantity is called **defuzzification**. There are a variety of *defuzzification methods*, ranging from empirical verification to heuristic approaches. Nest figure present three often used defuzzification methods: LoM - *Left of Maximum*, MoM - *Mean of Maximum*, and RoM - *Right of Maximum*.



When choosing a defuzzification method, we can choose either method that determines the value of the action variable by calculation as the best compromise (*center of gravity* methods) or methods that search for an acceptable solution (methods of the *most significant maximum*).

📝 2.1.8

Often used defuzzification methods are:

- Left of Maximum
- Mean of Maximum
- Right of Maximum
- Center of Gravity

Fuzzy expert system

Chapter 3

3.1 MATLAB + Fuzzy Logic Toolbox

🕮 3.1.1

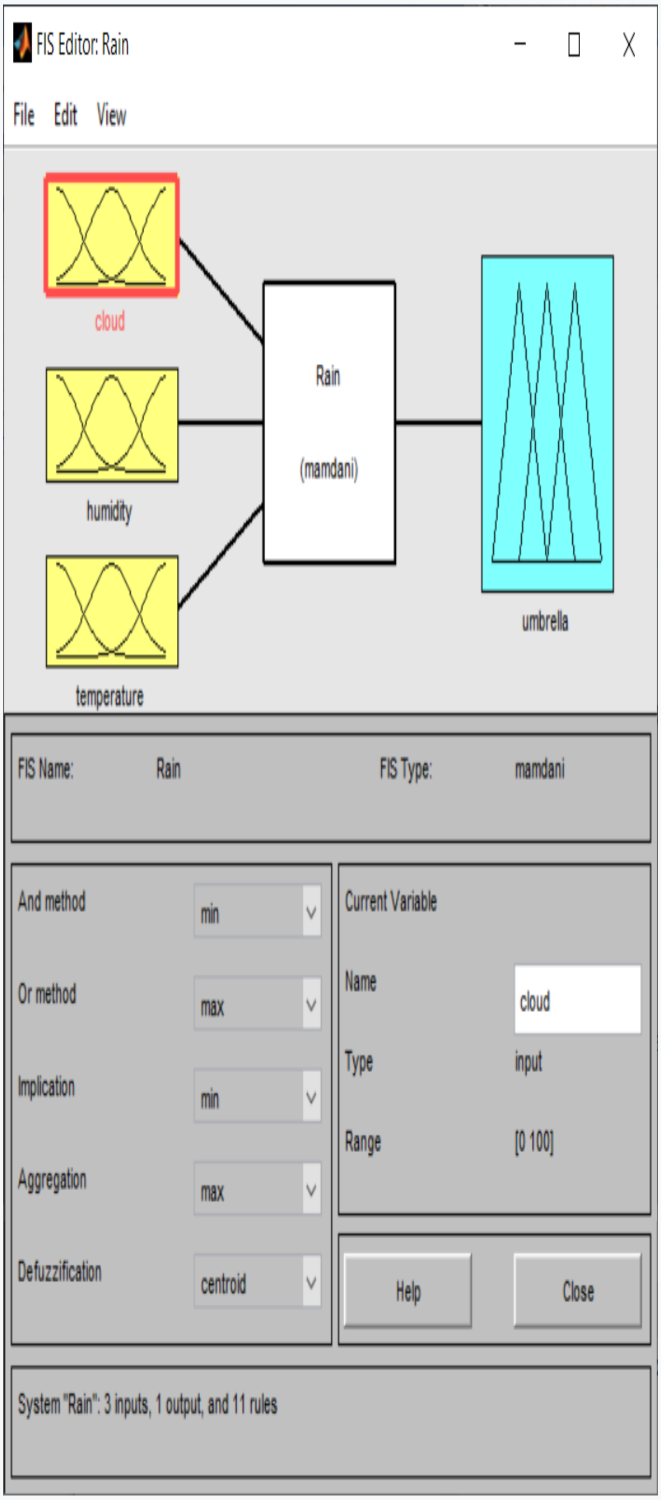
To store the expert knowledge in the form of IF-THEN rules and use this knowledge we construct **Fuzzy Logic Expert Systems.**

To model this expert systems we will use the MATLAB + Fuzzy logic toolbox environment.

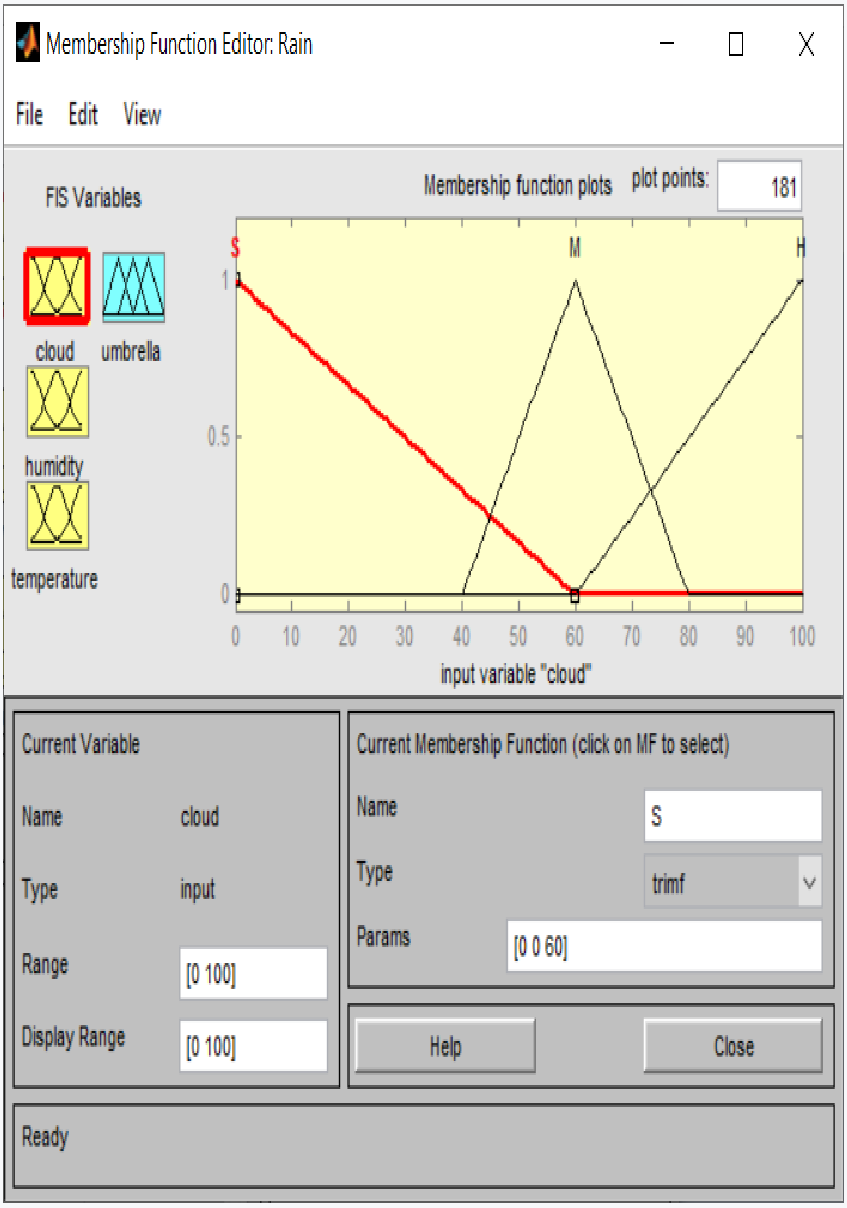
To start the support system in MATLAB we use command:

> fuzzy

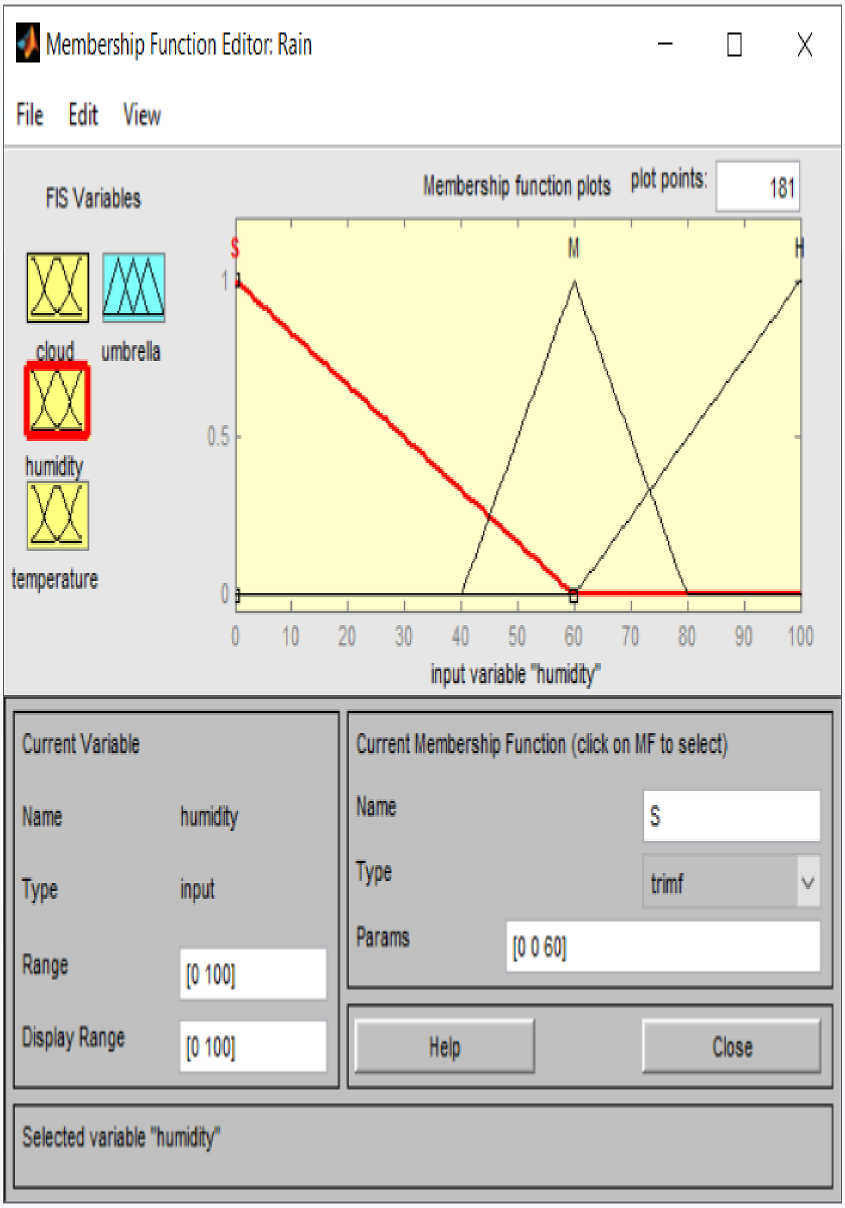
To present some easy example, let us to define expert system, which, based on the knowledge of humidity [%], cloudiness [%] and temperature [°C], will answer the question if you may to take the umbrella.



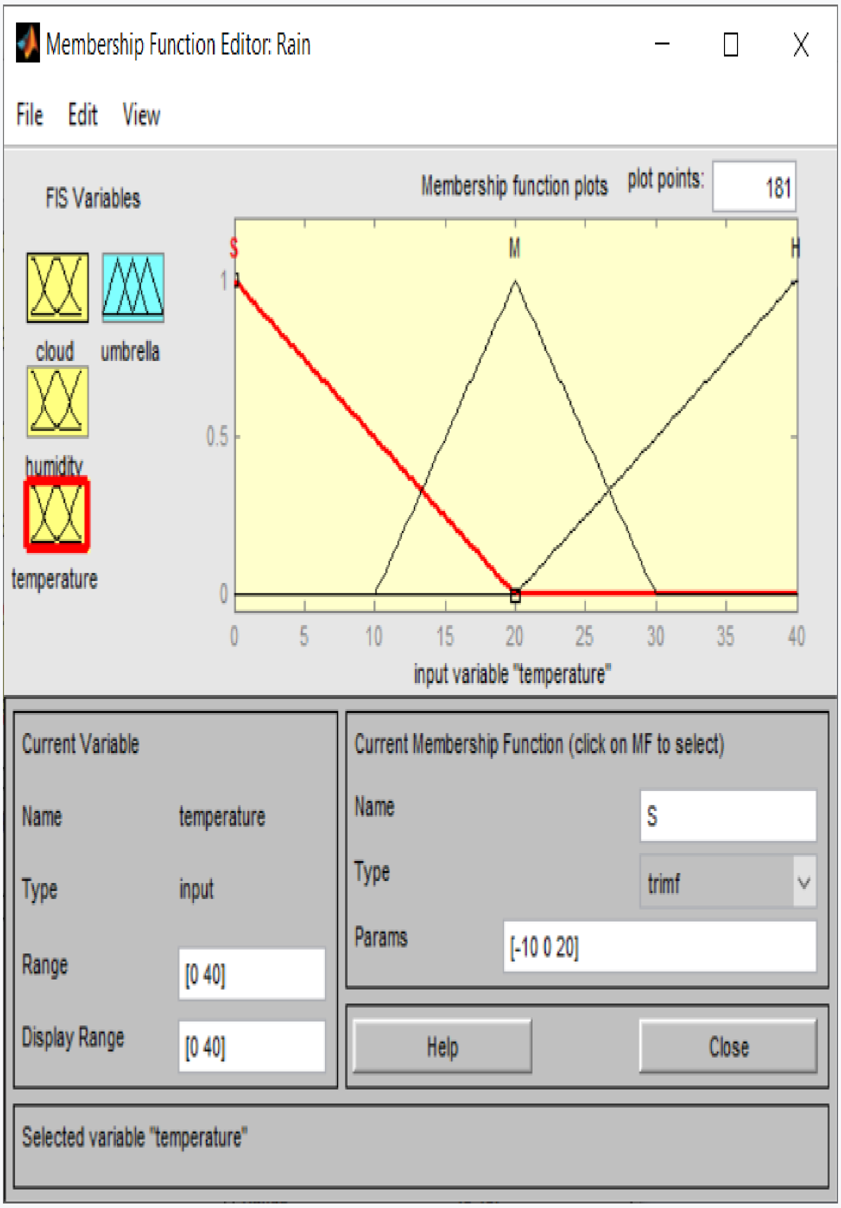
Firstly we define the membership function. For cloudiness we will define 3 fuzzy values (S - small, M - medium and H - high) in the range <0; 100> using the Membership Function Editor:



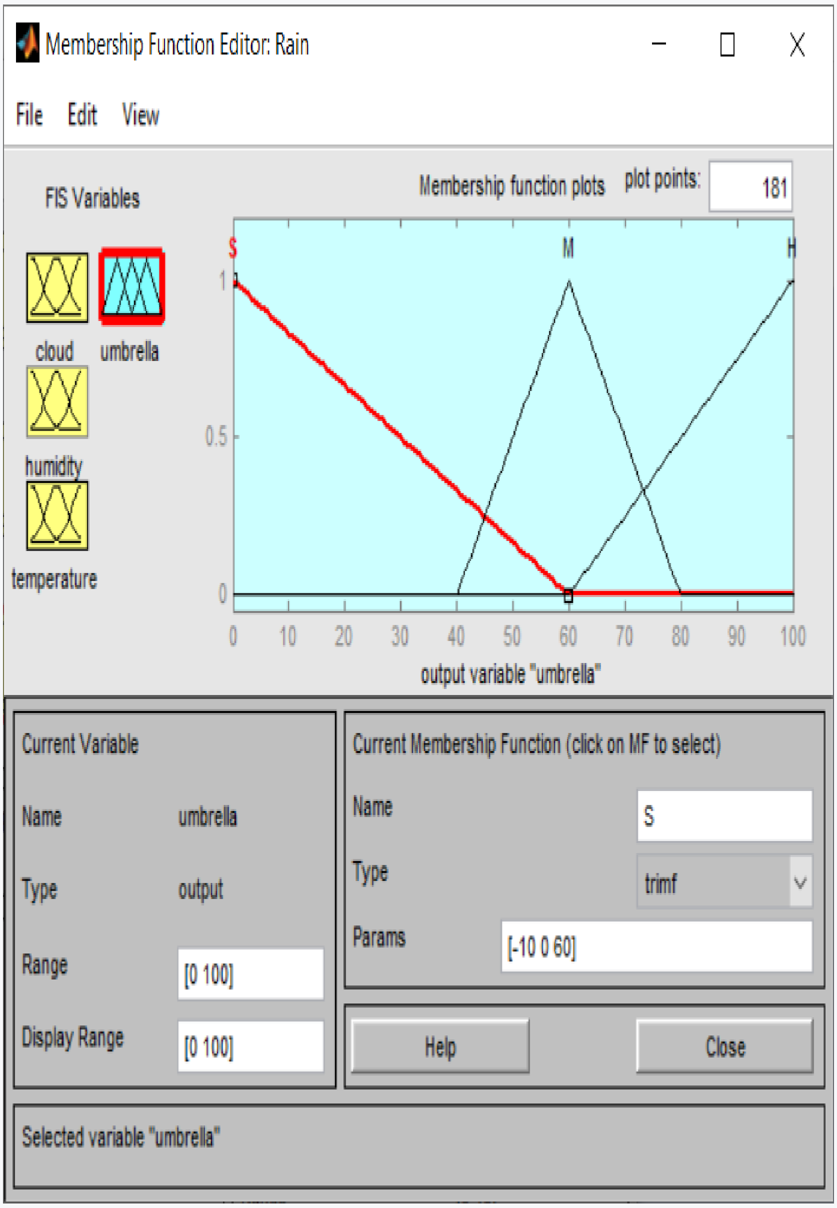
For humidity we will define 3 fuzzy values (S - small, M - medium and H - high) in the range <0; 100> using the Membership Function Editor:



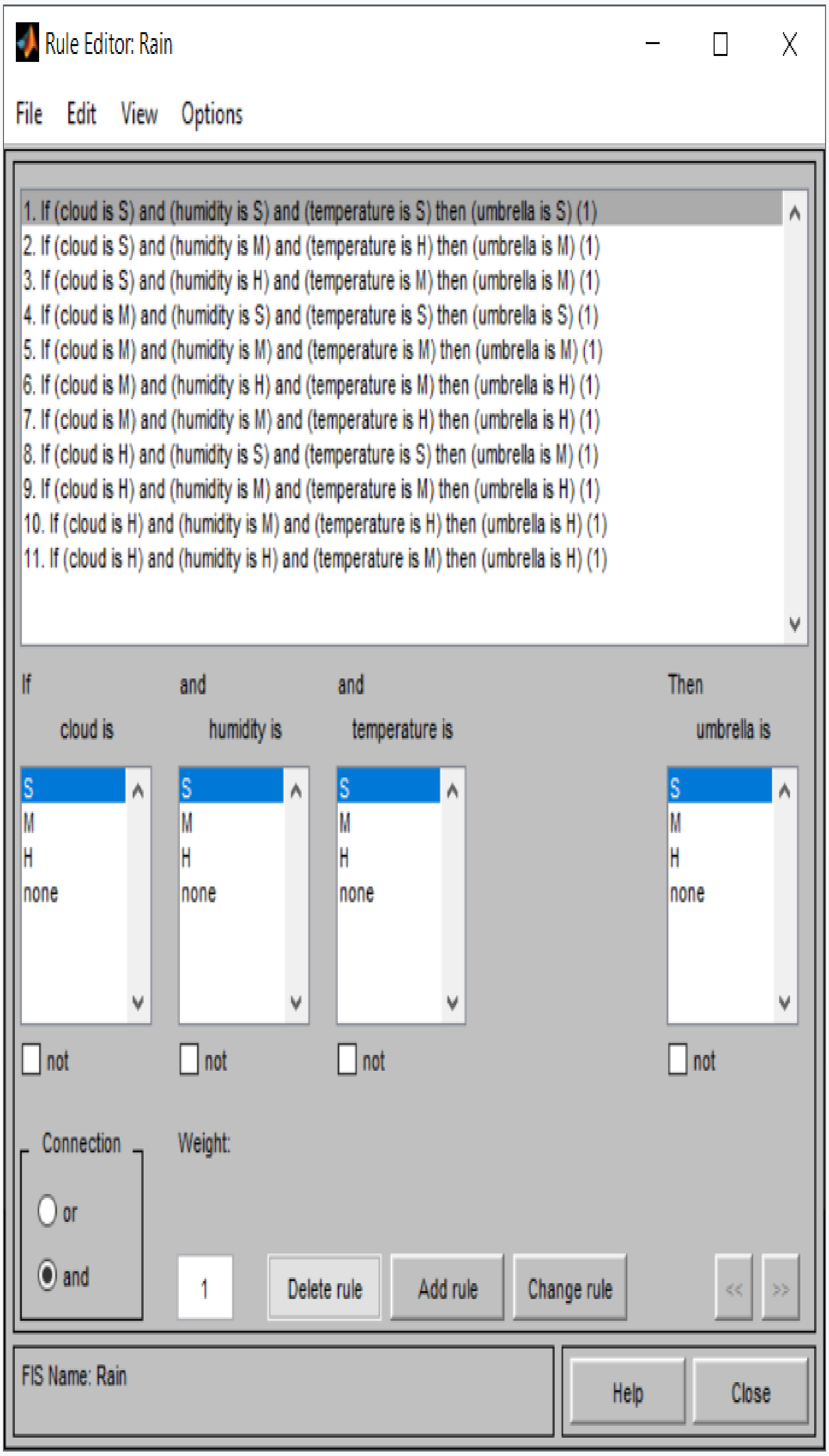
For temperature we will define also 3 fuzzy values (S - small, M - medium and H - high) in the range <0; 40> using the Membership Function Editor:



For output value we will also define 3 fuzzy values as probability to take umbrella (S - small, M - medium and H - high) in the range <0; 100> using the Membership Function Editor:

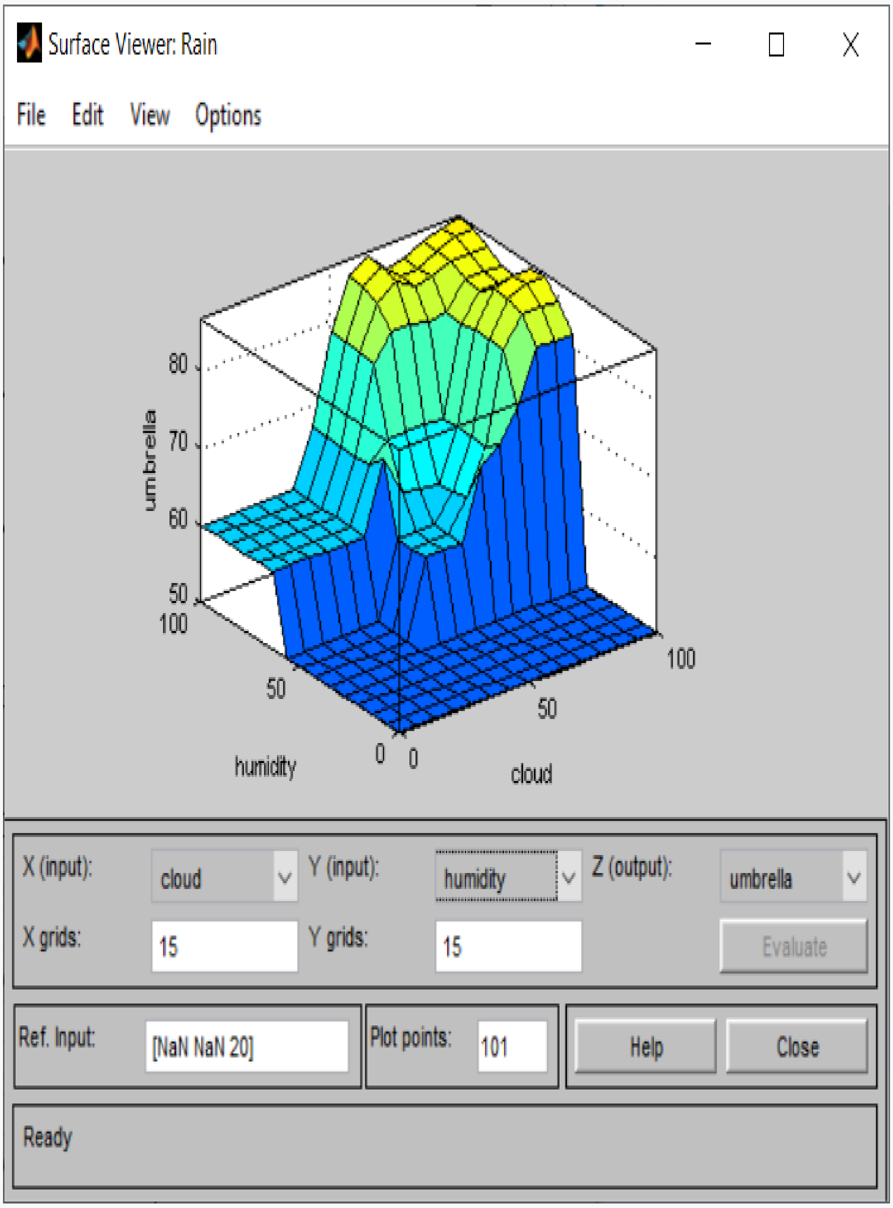


Based on this parameters we can now define the IF-THEN rules:



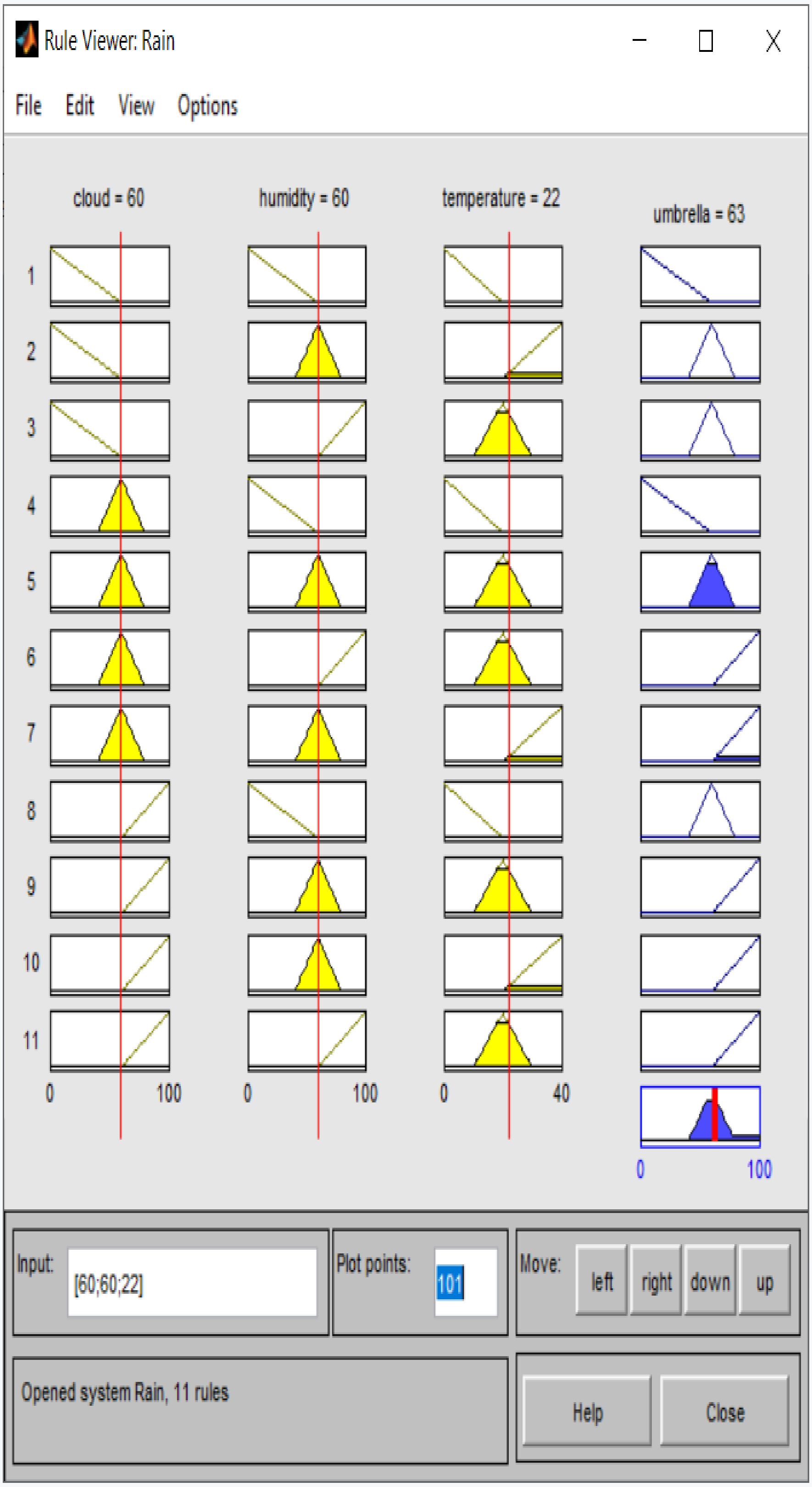
You can see, that there are not defined rules for all combinations of input parameters. This is typical situation when collecting knowledge from concrete experts. Some combinations cannot occur, with some the expert has no experience. We can reduce this problem by combining the knowledge of several experts.

The interaction of the entire set of rules can be well visualized using surface as the interdependence of two input parameters:



This surface can also alert us to inconsistencies in expert opinion or some missing rules.

Once the knowledge base of the expert system is created, we can use it to draw conclusions in specific situations. The Rule Viewer presents it very well. For concrete input (60, 60, 22) it resents not only the result (probability to take the unbrella is 63%), but also presents which rules has been activated and how they contribute to the overall result:



:

Final fuzzy expert system could be stored as a file, for example Rain.fis for further use. This file is standard text file, so you can edit it:

```
[System]
Name='Rain'
```

```
Type='mamdani'
Version=2.0
NumInputs=3
NumOutputs=1
NumRules=11
AndMethod='min'
OrMethod='max'
ImpMethod='min'
AggMethod='max'
DefuzzMethod='centroid'

[Input1]
Name='cloud'
Range=[0 100]
NumMFs=3
MF1='S':'trimf',[0 0 60]
MF2='M':'trimf',[40 60 80]
MF3='H':'trimf',[60 100 140]

[Input2]
Name='humidity'
Range=[0 100]
NumMFs=3
MF1='S':'trimf',[0 0 60]
MF2='M':'trimf',[40 60 80]
MF3='H':'trimf',[60 100 140]

[Input3]
Name='temperature'
Range=[0 40]
NumMFs=3
MF1='S':'trimf',[-10 0 20]
MF2='M':'trimf',[10 20 30]
MF3='H':'trimf',[20 40 50]

[Output1]
Name='umbrella'
Range=[0 100]
NumMFs=3
MF1='S':'trimf',[-10 0 60]
MF2='M':'trimf',[40 60 80]
MF3='H':'trimf',[60 100 120]

[Rules]
```

```
1 1 1, 1 (1) : 1
1 2 3, 2 (1) : 1
1 3 2, 2 (1) : 1
2 1 1, 1 (1) : 1
2 2 2, 2 (1) : 1
2 3 2, 3 (1) : 1
2 2 3, 3 (1) : 1
3 1 1, 2 (1) : 1
3 2 2, 3 (1) : 1
3 2 3, 3 (1) : 1
3 3 2, 3 (1) : 1
```

3.1.2

Define your problem based on three parameters and construct the fuzzy-expert recommender system to solve it. Was it easy?

Decision systems modelling

Chapter 4

4.1 Generalized algorithm for modeling decision systems

📖 4.1.1

When modelling decision systems on a computer, it is necessary to have a system that enables knowledge acquisition from the reactions of the IS users when retrieving information. It primarily concerns the simulation of decision-making processes as well as ways of knowledge acquisition.

Generally, human mental behaviour may be featured as decision-making, planning, coordinating, and communicating, being based on information acquisition, storage, evaluation, and classification. In order that any system could substitute or "support" a human in some of the above-mentioned activities, such a system must be primarily able to communicate well with the human as well as to offer him/her tools, which would directly support the activities. Other important features are:

- possibility to apply heuristics (together with formally expressed knowledge),

- the capability to provide explanations concerning derivations-in-progress as well as concerning the applied knowledge,

- possibility to simply integrate a new knowledge increment into the existing sum of the system knowledge.

Simulation of decision-making processes is featured as follows:

- decision-making is not based on analytic information only, but predominantly on the knowledge represented by both cognitive and abstraction processes (which are the privilege of brain activities);

- decision making can be done through various approaches depending on the number of judging persons;

- it is very difficult to formulate an algorithm for the decision-making procedure;

- a lot of information used during decision-making is of external origin with respect to the already implemented data base of the decision-making problem.

A decision-making process can be defined as an organic unity of three phases:

- information (knowledge acquisition);

- planning (considering alternatives);

- selection (variant selection).

In order to identify the structure of a decision-making process and thus establish prerequisites to find effective procedures for its algorithmization, we have to deal with decision-making processes from a wider, especially methodological point of view.

A generalized algorithm for decision system modeling by uncertainty has been presented in [Klimes 2011].

4.1.2

A decision-making process can be defined as an organic unity of three phases in the order:

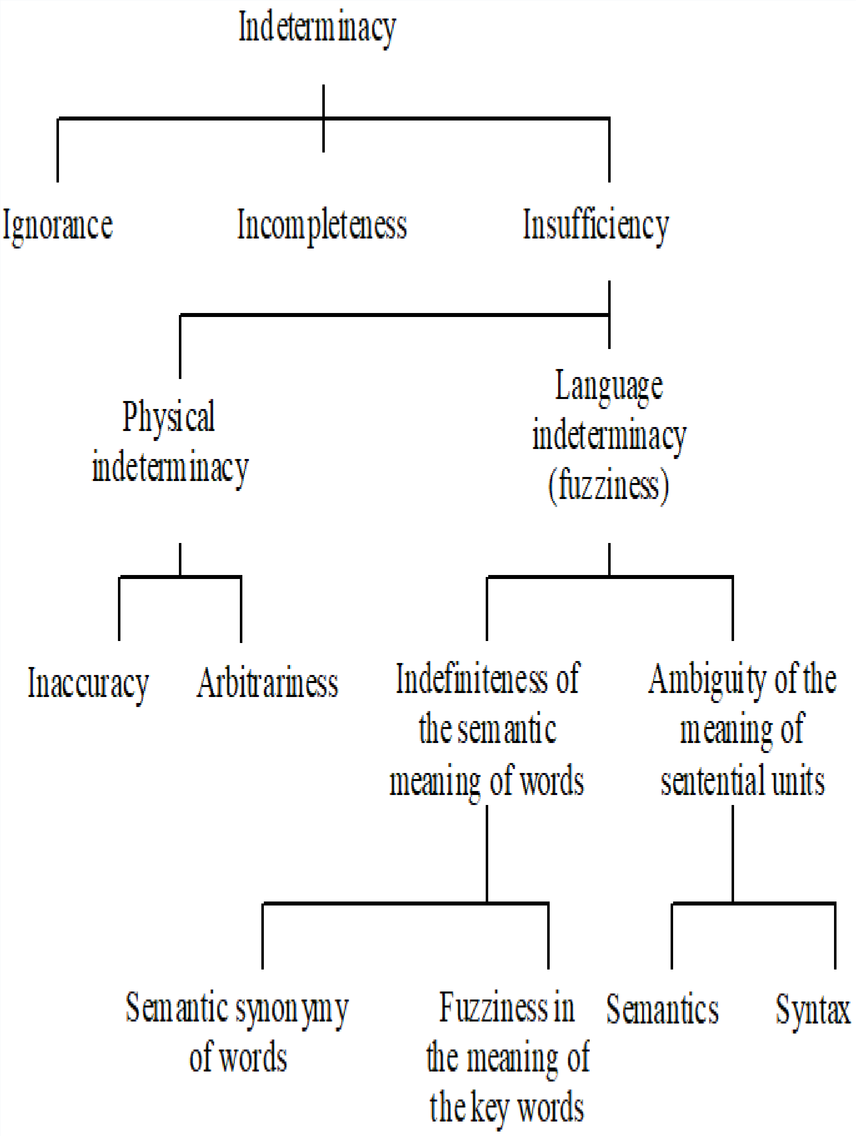
- information (knowledge acquisition);
- selection (variant selection);
- planning (considering alternatives);

4.2 Uncertainy in decision systems

4.2.1

One of the characteristic features of decision-making processes is the fact that they often work with indeterminate and non-metric information, which often stems from the fact that the input quantities of these processes are entered by a human based on his/her experience, opinion, etc.

Indeterminacy, or insufficient definition of these processes, also has its own structure enabling the use of corresponding tools for work with a corresponding type of incompleteness. The basic structure of incompleteness in the area of information (we are primarily interested in this area) can be depicted by the following scheme:



When analyzing indeterminacies occurring during solving a given decision-making process, it often comes out that the indeterminacy is, in fact, represented by insufficient information caused either by external factors (physical indeterminacy) or a language, by means of which a human factor enters the solution. The insufficiency itself is primarily represented by physical indeterminacy caused either by inaccuracies in measuring given quantities and their quantitative expression or in the existence of physical possibilities with a more or less accidental occurrence that is impossible to be predicted in advance with sufficient accuracy. The second type of insufficient information is the use of a natural language, introduced to the decision making process by a human, who then describes the decision-making process itself and its functioning. Insufficiency of this information consists in the fact that the human factor is forced to use a finite number of words in a finite time to describe situations, which, in fact, can be of an infinite number. This fact necessarily leads to the fact that most of the words, as well as sentential units, have a considerable variance in their own meaning.

These indeterminacies (indefinitenesses) of the semantic field of words are caused both by semantic synonymies of words and (primarily) by certain fuzziness in the meaning of the keywords. The fuzziness then becomes the key cause of the fact that classical mathematics, and exact sciences in general, were not adequately capable enough to work with linguistically defined situations. There was a change in the last period when so-called fuzzy mathematics was created, enabling it to work efficiently with such verbally described situations.

4.2.2

One of the characteristic features of decision-making processes is the fact that

- they often work with indeterminate and non-metric information...
- they often work with intercontinental and non-deterministic information...
- they often work with interconnections and non-commercial information...

4.3 Decision process model

4.3.1

Despite the fact that the decision-making process involves numerous indeterminacies, its structure can be defined relatively well. Especially, the elements of this decision-making process can be divided into the following groups:

S – a set of situations,

D – a set of all possible solutions,

G – a set of all targets (admissible) for further functioning of a given system,

F – a set of all degrees of existence (probabilities) of a given object,

K – a set of all evaluations for a given solution,

T – time interval.

The decision-making process itself is represented by various mappings among these sets. It particularly concerns the following mappings:

1. the process of information completion about a given situation and its evaluation, i.e. selection of only the information which has importance for the final solution:

$$M_1 : S \times T \times F \to S \times T \times F$$

2. the process of creating a set of admissible solutions, which consists of two partial processes

$$M_2 = M_{22} \circ M_{21}$$

where:

M_{21} – formulating management targets based on the description of a given situation,

M_{22} – formulating admissible solutions:

$$M_{21}: S \times T \times F \to G \times S \times T \times F$$

$$M_{22}: G \times S \times T \times F \to D \times S \times T \times F$$

3. the process of modelling effects of admissible solutions

$$M_3 : D \times S \times T \times F \to D \times S \times T \times (S \times T)^* \times F$$

where $(S \times T)^*$ defines the set of all chains over $(S \times T)$,

(where each admissible solution is allocated with a set of situations including their time courses, which arise from the given decision.

4. the process of acceptance of the solution itself, which consists of two partial processes

$$M_4 = M_{42} \circ M_{41}$$

where:

M_{41} – evaluating the behaviour of effects of admissible solutions,

M_{42} – selection of the best variants:

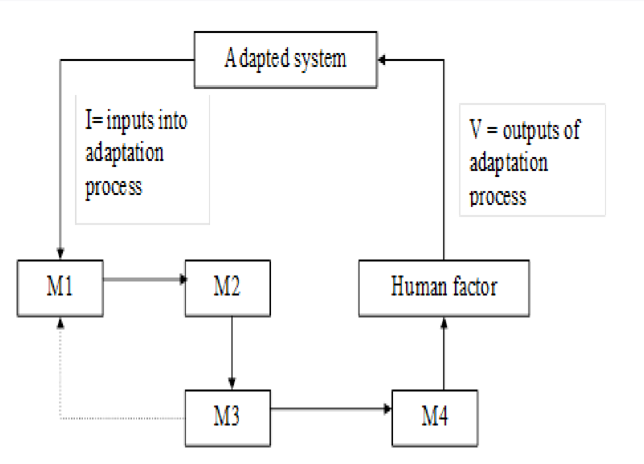
$$M_{41} : D \times S \times T \times (S \times T)^* \times F \rightarrow D \times K \times T \times F$$

$$M_{42} : D \times K \times T \times \rightarrow D \times T$$

The whole decision-making process is created through the gradual composition of these partial processes

$$M = M_4 \circ M_3 \circ M_2 \circ M_1$$

As we can depict in the following diagram



Let us note that the realization of particular processes M_1 – M_4 can be ensured by means of so-called fuzzy algorithms using the results of fuzzy sets theory.

Particular processes M_1 – M_4 differ one from another by the character of input and output quantities as well as by other relations being performed within the process.

4.3.2

The decision-making process can be divided into the following groups:

- S – a set of situations,
- D – a set of all possible solutions,

- G – a set of all targets (admissible) for further functioning of a given system,
- F – a set of all degrees of existence (probabilities) of a given object,
- K – a set of all evaluations for a given solution,
- T – time interval,
- A – a set of alfanumeric symbols representing the possible solutions.
- B – a set of bad decisions,
- C – a set of certain coins needed to generate the values of a random variables,

4.3.3

Generally speaking, each process can be featured by the following sextuplet

$$(T_i, T_v, X_{ch}, T_p, T_k, T_{kk})$$

where

T_i – type of inputs,

T_v – type of outputs,

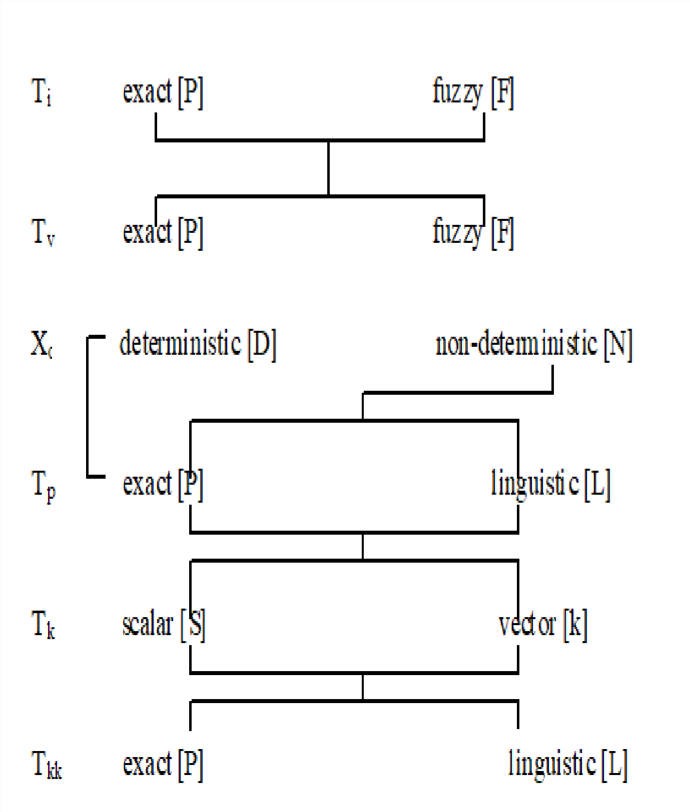
X_{ch} - character of outputs,

T_p – type of indeterminacy,

T_k – type of selection criterion,

T_{kk} – type of particular elements of the criterion.

The particular types may take the values given in the following diagram:



When creating the simulation system, we assumed that it would not be necessary to deal with all processes $M_1 \ldots M_{41}$, M_{42} because some of them are evident.

We can presume that the process of information completion about a given situation and its evaluation M_1 is a part of the acquisition of the input situation evaluation and it is not necessary to explain it in more details.

The key processes in the decision-making process can be considered processes

M_{22}, M_3, M_{41}, M_{42}.

4.3.4

The key processes in the decision-making process can be considered processes:

- M22, M3, M41, M42.
- MM, MN, MO, MI, MP, MPS.
- A, B, C, D, E, F, G, H, I, J.
- M1, S2, T3, K4, U1, U2, U3.

4.4 Model of web system adaptation

🕮 4.4.1

The adaptive web system monitors the behaviour and characteristics of an individual user who is then offered adapted information. The basic motivation to create the adaptive web system is the variability of individual users. Thus it is necessary to prepare information corresponding to their abilities, preferences, and needs. We can adapt the user interface, information content and structure, and other qualities of the provided information. It implies that the users of the adaptive system cannot be anonymous. The adaptive system keeps the information about individual users which is continually evaluated and updated.

The input information into a decision-making process on the selection of the most important information visualised on the web can be divided into the following groups:

- data volume;
- information content;
- importance;
- availability;
- credibility;
- the cost of information acquisition;
- time for information acquisition;
- size from the point of view of space taken up on disks, etc.

The output of the whole system should be a set of all possible solutions *D* represented by the following characteristics:

- suitability of the provided information, expressed, e.g., by:

+ real content,

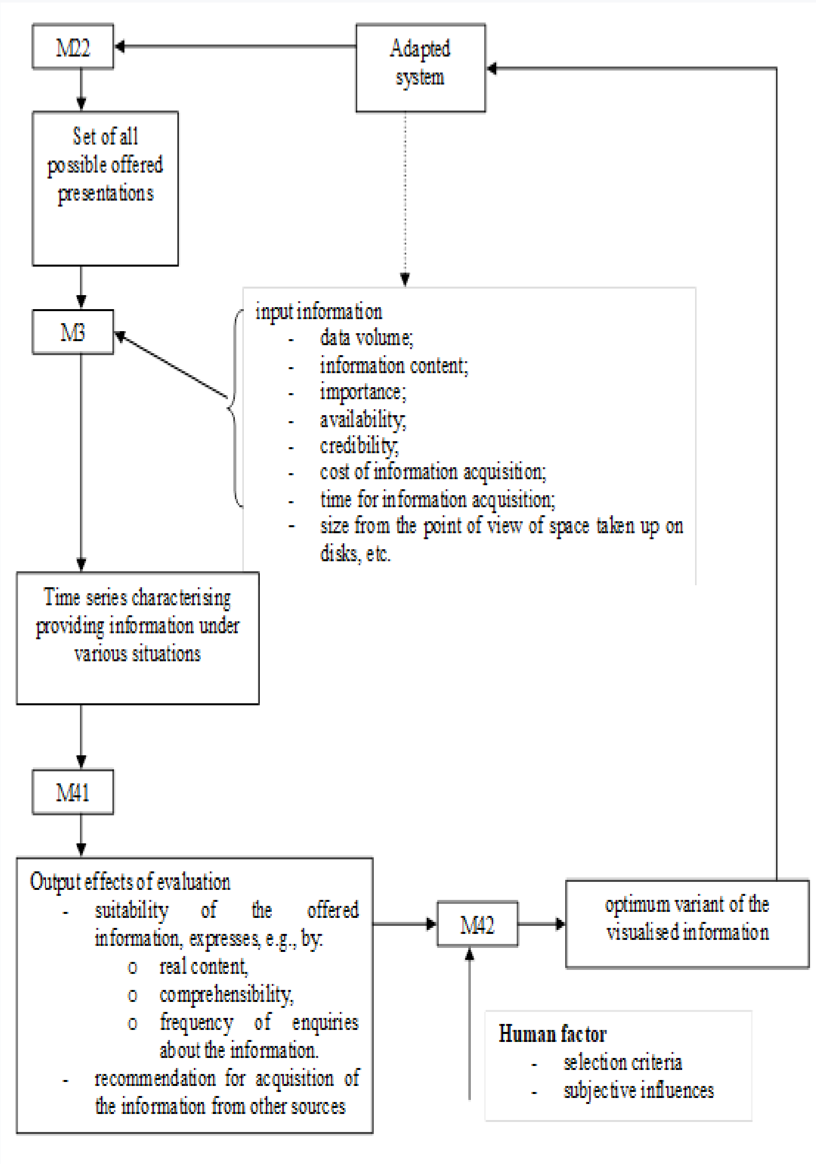
+ comprehensibility,

+ frequency of enquiries about the information,

- recommendation for the acquisition of information from other sources.

The following part will try to indicate how to perform particular processes while stemming from the fact that some parts of these processes, specific for conditions of retrieval on the web, have been already processed. This particularly concerns processes connected with M_3 – simulating the impacts of particular decisions.

Based on this specification, we can define the following general structure of the decision-making system solving a given problem:



4.4.2

What is the basic motivation to create the adaptive web system?

- The variability of individual users.
- The number of different parameters.
- The insufficient knowledge of the problem.

4.5 Model of choosing an optimal option of the mapped information on the web

4.5.1

First, let us consider the M_{42} process. From the classification point of view, as depicted in the process types figure, the process of selecting the optimum variant belongs to the following two key categories:

I. category = (P, P, D, P, V, P)

II. category = (F, F, N, L, V, L)

The two categories correspond to the fact that during the selection of the optimum variant, the input quantities are entered either exactly or verbally, the output quantity is either exact (detailed analysis of the visualization variant) or on the contrary as a verbally described suitability of the given mapping, and a vector criterion entering the selection of mapping can also be described either defined exactly or set verbally.

It is evident that the realisation of both processes M_{42}^{I} and M_{42}^{II} is quite different. While in the case of M_{42}^{I} process, it concerns a classic vector optimisation, in the case of M_{42}^{II} process, the situation is quite different.

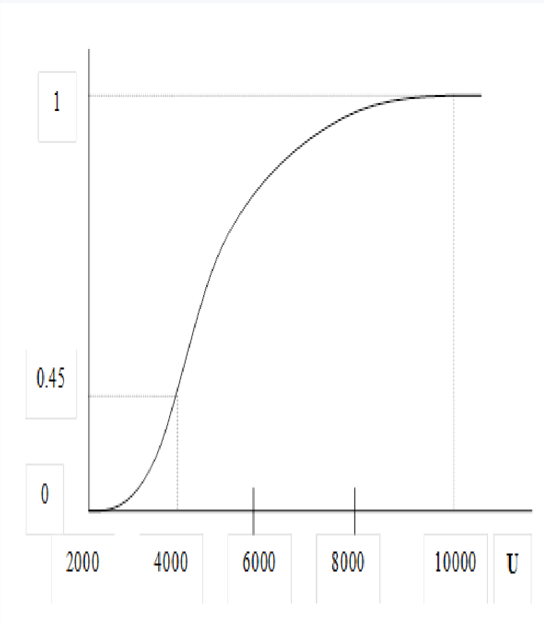
We stem from the fact that the inputs of the decision-making process M_{42}, as well as the decision-making algorithm itself, can be described uncertainly, primarily by linguistic terms characterising values of specific quantities or by relations between specific quantities. However, to make the performance of such inputs and algorithms computerised, it is necessary to use a suitable mathematical apparatus

One of the possibilities for how to perform such quantities is the fuzzy set theory. We will mention here some descriptions from this theory that are possible to be used in a decision-making process.

Let *U* be a set of objects, which are concerned in our decision-making process (e.g. *U* is a time interval, or an interval representing average costs to acquire information expressed, e.g., by the amount of database passes, etc.). The fuzzy set in *U* will be called the mapping

$$A : U \to [0,1]$$

where quantity $A(x),\ x \in U,$ is called by the membership level of the element *x* in *A*. As for each function, we can represent *A* by the guarantee of the function. For example, if *U= [2000, 10 000]* is the universe representing the quantity of information, then *A* in *U* is defined by the diagram in the next figure:



represents the verbal term: *A = HIGH INFORMATION LEVEL*.

The fact that *A* is a fuzzy set in *U* will be marked as $A \subseteq U$. Furthermore, the term “fuzzy relation” is important for our purposes. If U_1 *and* U_2 are two universes, then fuzzy relation is a fuzzy set in their Cartesian product, i.e. $R \subseteq U_1 \times U_2$. If e.g.

$U_1 = U_2 = U$ in the previous example, then we can define the fuzzy relation $R = NEARLY\ EQUAL\ to \subseteq U \times U$ by means of a functional prescription

$$R(x, y) = e^{-|x-y|}, x, y \in [2000, 10000]$$

Similarly to classical sets, we can define analogue operations within the class of fuzzy sets. Particularly, if *U* is a universe, $A, B \subseteq U$, then we define:

$$(A \cup B)(x) = \max\{A(x), B(x)\}$$

$$(A \cap B)(x) = \min\{A(x), B(x)\}$$

$$\neg A(x) = 1 - A(x)$$

$$(A \times B)(x, y) = \min\{A(x), B(y)\}$$

For our next objectives, it is important to introduce the term linguistic variable, i.e. the variable χ represented by the following system

$$\chi = \langle X, \tau, M \rangle$$

where *X* is a domain of values, τ is a set of terms (i.e. specific words) and *M* is semantics, i.e. representation assigning a fuzzy set $M(t) \subseteq X$ to each term *t*.

If we consider the linguistic variable

χ = *SIZES*.

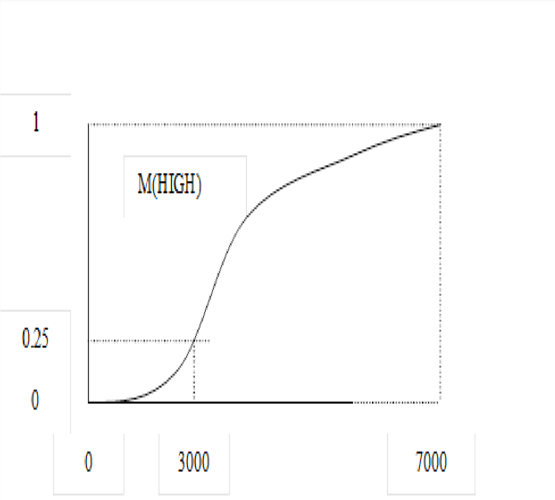
The set of terms *t* of this variable is created e.g.by words

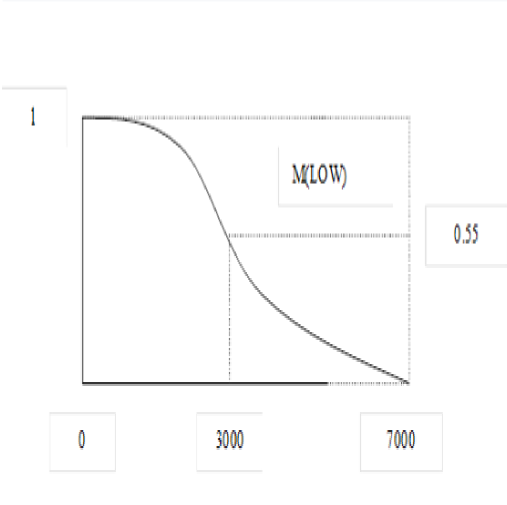
τ = {HIGH, LOW, MEDIUM, VERY HIGH, ...}

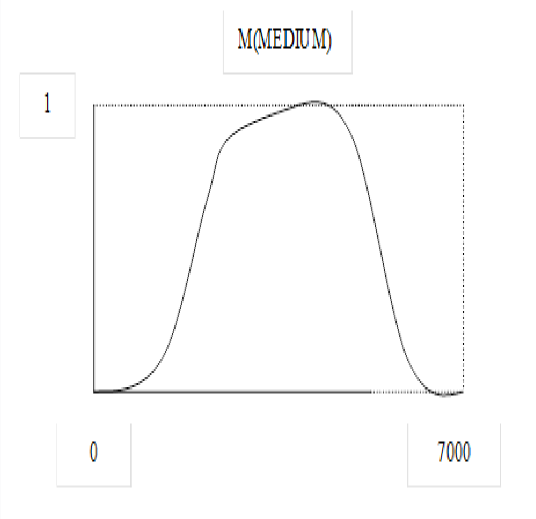
Then we can define as a domain of values, e.g., interval

X = <0, 700>

(e.g. *t* suitability of the offered information) and function *M* for particular terms can be defined e.g. in the following diagrams and relations:







and furthermore

$$M\,(\text{VERY } t)\,(x) = [M(t)\,(x)]^2\,;\, x \in X,$$
$$M\,(\text{NOT } t)\,(x) = 1 - M(t)\,(x),$$
$$M\,(t_1 \text{ AND } t_2)\,(x) = \min\,\{M(t_1)\,(x), M(t_2)\,(x)\},$$
$$M\,(t_1 \text{ OR } t_2)\,(x) = \max\,\{M\,(t_1)\,(x), M\,(t_2)\,(x)\}.$$

The,n for example, the suitability level of the offered information *x = 3000 t*

corresponds to the verbal expression

t = NOT VERY HIGH AND NOT VERY LOW

with the membership degree

M(t)(x) = min {M (NOT VERY HIGH) (x), M(NOT LOW) (x)}= min {1-M (VERY HIGH) (x), 1-M (LOW) (x)} = min {1-(M(HIGH)(x))^2, 1-M(LOW)(x)} = min {1-0.25^2, 1-0.55} = min {0.9375, 0.45} = 0.45

i.e. only a half.

📖 4.5.2

Using linguistic variables, we can set up fuzzy algorithms of certain processes.

If the input quantities of a given system are x=(x_1..x_n) and the output ones y=(y_1..y_m), then the fuzzy algorithm means the expression:

If $\varphi(x_1..x_n)$, **then** $s(y_1..y_m)$, **or** $y(y_1..y_m)$,

where $\varphi(x_1..x_n)$, $s(y_1..y_m)$ $y(y_1..y_m)$ are linguistic expressions concerning the individual mentioned quantities.

For example, in the process M_{42}^{II} we consider the following situation. One of the partial decision-making rules in this process can concern e.g. the relations among the importance (e), availability (v), unit costs for information acquisition (j), and the selection of the given mapping (t).

Let us suppose that $e \in \langle a_1, b_1 \rangle, v \in \langle 0,100\% \rangle$. Then the verbal expression of one of the decision-making rules can be as follows:

R_1: if e = MEDIUM, v = HIGH, j = MEDIUM, then t = HIGH SUITABILITY

or

R_2: if e = HIGH, v = HIGH, j = HIGH, then t = MEDIUM SUITABILITY.

We then deal with the relation among 4 linguistic variables

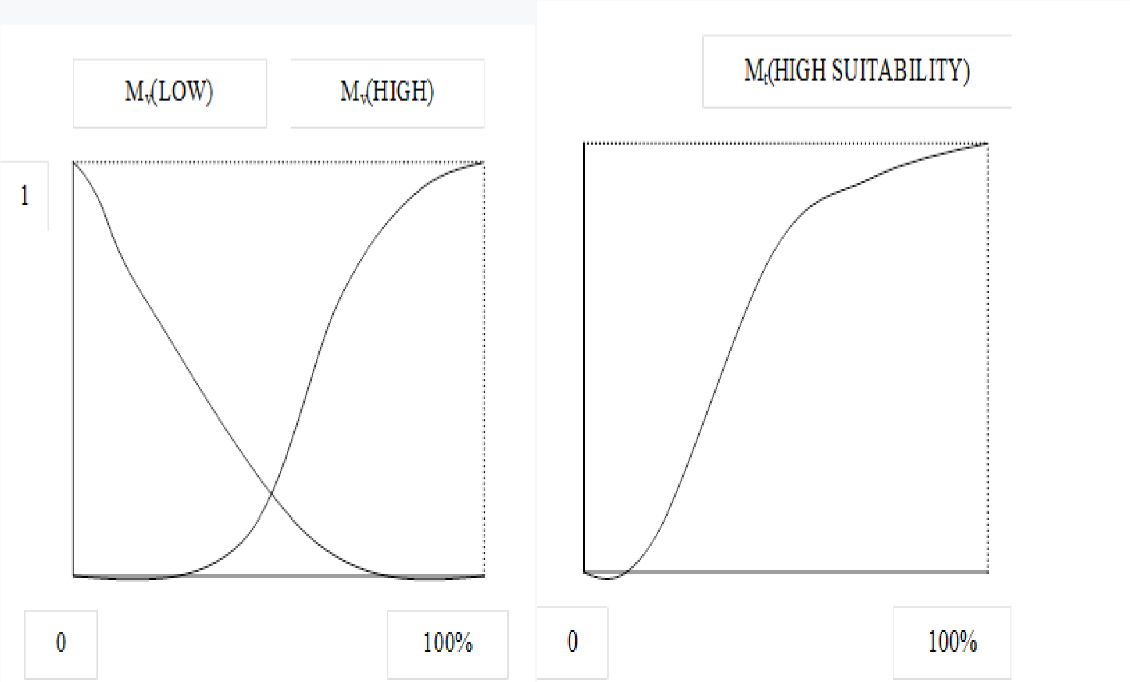
E = <{LOW, MEDIUM, HIGH, VERY, AND, NOT, OR}, <a_1,b_1>, M_e>

V = <{LOW, MEDIUM, HIGH, VERY, AND, NOT, OR}, <0, 100 %>, M_v>

J = <{LOW, MEDIUM, HIGH, VERY, AND, NOT, OR}, <a_2,b_2>, M_{1j}>

T = <{LOW SUITABILITY, MEDIUM SUITABILITY, HIGH SUITABILITY, VERY, AND, NOT, OR}, <0,100%>, M_t>,

some fuzzy sets of which can be, e.g., as follows:



Generally, we can say that we possess such rules

R_1: if e = A_1, v = B_1, j = C_1, then t = D_1

R_k: if e = A_k, v = B_k, j = C_k, then t = D_k.

In case we have a specified input vector (e,v,j), we can determine the corresponding value with a corresponding mapping t as follows.

Let d_i be an x-coordinate, the gravity center of the surface lying above the graph of function D_i, i.e.

$$d_i = \left(\int_{x \in <0,100>} x.D_i(x)dx\right) / \int_{x \in <0,100>} D_i(x)dx); \; i=1, \ldots, k$$

Let furthermore

$$s_i = \min \{A_i(e), B_i(v), C_i(j)\} \in \langle 0,1 \rangle$$

Then we can put

$$SUITABILITY(t) = \frac{\sum_{i=1}^{k} d_i . s_i}{\sum_{i=1}^{k} s_i} .100 \in \langle 0,100\% \rangle$$

4.5.3

The set of rules R_i,...R_k can be preferably obtained by using expert assessments.

One of the problems connected with the application of fuzzy sets in process M_{42} is the problem of constructing the corresponding fuzzy sets *M(t)*, where *t* denotes the terms of the particular linguistic variables. We will show here several possible approaches to solving the problem.

Let us consider e.g. a linguistic variable *V*=INFORMATION AVAILABILITY and its term *t* = HIGH. We need to define functions M_v (HIGH) : [0,100] → [0,1].

(1) We have *m* available experts. For any value $x \in [0, 100]$, the experts answer the question of whether the value corresponds to the expression HIGH or not. Let *n* out of these experts confirm that it corresponds, then

$$M_v(HIGH)(x) = \frac{n}{m} \in [0,1]$$

(2) Le us assume again that we have *m* experts and only values *x = 0, 1, 2, ..., 100* are being tested. (Note: We will use the principles, described in the part focused on

Multi-criteria Decision Analysis as Saaty's method.) Each of the experts then defines values m_{ij} in such a way that:

M_{ij} = 1, if he/she considers M_v (HIGH)(i) approximately equals to M_v (HIGH)(j)

M_{ij} = 3, if he/she considers M_v (HIGH)(i) is a little bigger than M_v (HIGH)(j)

M_{ij} = 5, if he/she considers M_v (HIGH)(i) is bigger than M_v (HIGH)(j)

M_{ij} = 7, if he/she considers M_v (HIGH)(i) is a bit bigger than M_v (HIGH)(j)

M_{ij} = 9, if he/she considers M_v (HIGH)(i) is much bigger than M_v (HIGH)(j)

If it was defined already m_{ij}, $i < j$,

it is put $m_{ji} = 1/ m_{ij}$.

If the maximum inherent number of the matrix $A = \|m_{ij}\|$, we can find the solution x = $(x_1..x_{100})$ of the matrix equation

$$(A - \alpha.E).X = 0$$

Then we put

$$M_v(HIGH)(i) = \frac{x_i}{\sum_{j=1}^{100} x_j}$$

The same situation is also in the case of process M_{41}, which can be also decomposed into two parts M_{41}^{I}, M_{41}^{II}. For the deterministic part of M_{41}, we can use classical methods for analyzing time series, which are usually available.

4.6 Application

4.6.1

Working with a decision-making modelling system might be divided into two separate parts as follows:

a) creating a model, i.e. constructing appropriate fuzzy sets and establishing decision-making rules. This stage is relatively demanding as the analytic activities require close cooperation with experts.

b) applying the model and its recovery, i.e. based on specific input parameters pursuing the calculations of the input values. This stage is demanding as simple mathematic operations, are mostly done by computers.

Many applications of the presented algorithm have been published. For example:

Klimes, C. Model of adaptation under indeterminacy. *Kybernetika*, Volume 47 (2011), Number 3, Pages 356 – 369. Click here for full text.

Klimes, C. & Bartos, J. IT/IS security management with uncertain information. *Kybernetika*, Volume 51 (2015), Number 3, Pages 408 – 419. Click here for full text.

4.6.2

Did you get familiar with some published examples of the presented model application?

Multi-Criteria Decision Analysis

Chapter 5

5.1 Multi-Criteria Decision Analysis

🕮 5.1.1

Multi-criteria decision analysis is a special algorithm that can be used for special kinds of recommender systems or decision support systems.

It is based on the selection of the most suitable alternative (variant) from a set of possible alternatives. These alternatives can be evaluated from the point of view of several different criteria. The evaluation of alternatives according to different criteria has a different range of values as well as a physical dimension. Some criteria have the character of maximization, others minimization. Criteria cannot be aggregated into one criterion.

In its applications, the knowledge of experts is appropriately used. Typical applications are for example: selecting the best software tool for our purposes, selecting the best family car, finding the best place for the new factory, etc.

🕮 5.1.2

Typical steps for performing Multi-Criteria Decision Analysis:

1. Structuring the decision problem
2. Specifying criteria
3. Measuring alternatives' performance
4. Scoring alternatives and weighting the criteria
5. Applying scores and weights to rank alternatives
6. Supporting decision-making

🕮 5.1.3

The first step is the determination of alternatives (variants) to the solution. It is necessary to determine all alternatives corresponding to the basic conditions for the solution:

- applications for the selection procedure,

- searching for all available alternatives.

The second step is establishing criteria for evaluating alternatives:

- we are preferring aggregate criteria,
- criteria must cover all important aspects of the problem,
- criteria must be independent and must evaluate all significant aspects of the problem.

We can use expert knowledge also in the first steps of problem-solving.

5.1.4

Typical order of main steps of decision-making analysis is:

- Applying scores and weights to rank alternatives
- Supporting decision-making
- Specifying criteria
- Measuring alternatives' performance
- Scoring alternatives and weighting the criteria
- Structuring the decision problem

5.2 Weighting the criteria

5.2.1

This step is typically started by the formation of an expert group. It is important to get representation from all important user groups. At the same time, it is not appropriate for the representatives of some groups to be represented in a significantly larger number than others. There would be an inappropriate assessment shift.

To determine the significance of the criteria we have more different methods, most often used are:

- Ordering criteria,
- Sorting criteria,
- Fuller's triangle,
- Saaty's method.

Based on the selected method we will ask every expert for his opinion and calculate the importance of every criterion.

📖 5.2.2

Ordering criteria

Ordering criteria is a very easy method. Each expert determines a clear order of importance of the criteria.

This evaluation will be stored in the table:

		criteria		
expert		1	...	j
	1			
	...			
	i			α_{ij}
	$\sum_i \alpha_{ij}$			

The significance of every criterion will be calculated:

$$B_j = 1 - \frac{\sum_i \alpha_{ij}}{\sum_j \sum_i \alpha_{ij}}$$

This method is very easy, but has some disadvantages:

- it is too strict, an expert cannot evaluate two criteria in the same order
- Distance between criteria is constant, expert cannot evaluate some criterion as much more important than any other criterion.

5.2.3

Scoring criteria

Experts will evaluate every criterion by a point value from a given range with the possibility of repetition.

		criteria			
expert		1	...	j	$\sum_j \beta_{ij}$
	1				
	...				
	i			β_{ij}	

This evaluation could be converted to weights for individual experts:

$$p_{ij} = \frac{\beta_{ij}}{\sum_j \beta_{ij}}$$

Next could be determined the significance of every criterion:

$$B_j = \sum_i p_{ij}$$

Some questions need to be answered. First is the problem of the appropriate choice of point range:

- we need a large enough range (twice the number of criteria could be appropriate),
- too large scale will be a problem for experts, they can only use a part of it (for example 100 points for 3 criteria),
- mostly "natural" ranges (1 ÷ 5, 1 ÷ 10, ...) are suitable.

The second question is about incorporating a null value. Usually, it is not included. Every criterion has some importance.

The danger of using part of the scale range is eliminated by conversion to weights.

📖 5.2.4

Fuller's triangle

Fuller's triangle (Triangle of pairs) is very easy for experts to use:

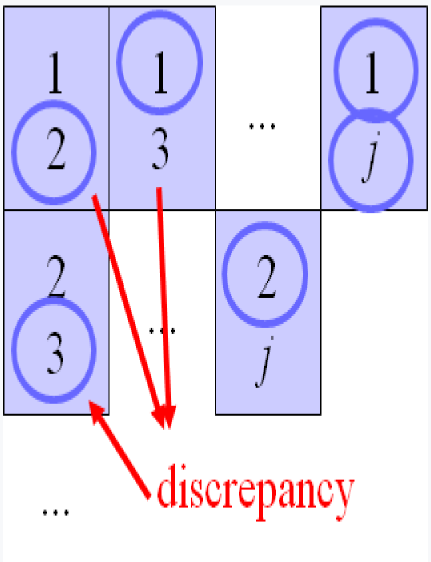
- we will compile all pairs of criteria,
- the expert determines the more important of the pair only,
- it is possible to leave the pair equally significant.

Evaluation by an expert:

- more significant of the pair = 1 point,
- both equally significant = 0.5 points.

A typical application of Fuller's triangle - each expert receives a list of all pairs of criteria and circles the more significant one in each pair. If he considers both criteria equally important, he circles both.

Unfortunately, the expert can commit an inconsistency in the assessment, as shown in the figure:



If we want to detect this discrepancy, we can use a simple application from Graph Theory that was presented in [Farana 2016]>

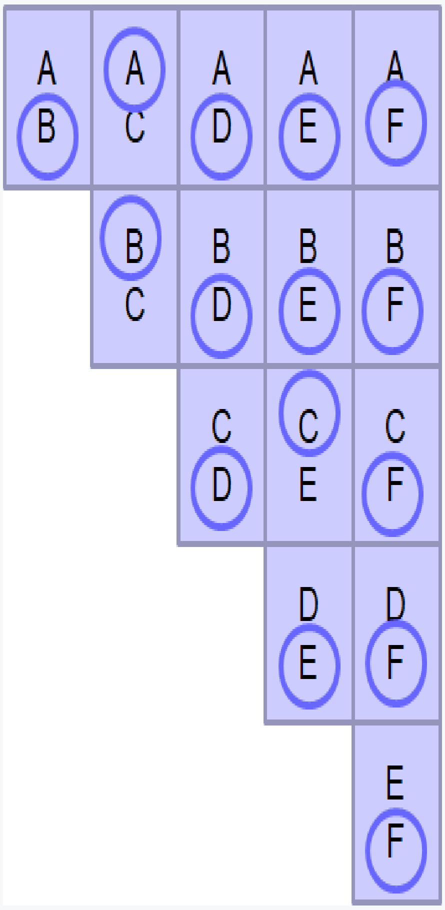
We will use a graph (complete graph) to describe the criteria>

- Vertices represent criteria.
- Edges represent individual ratings.
- We orient the edges to a more significant criterion.

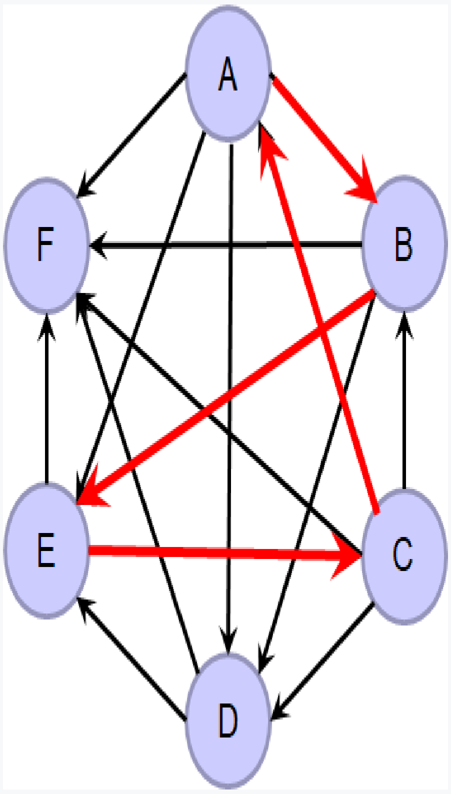
If the resulting graph is acyclic, no conflict has occurred.

We will apply acyclicity checking by numbering the vertices from start (with zero input degree) to the end (with zero output degree) vertex.

An example of conflicts in expert evaluation:



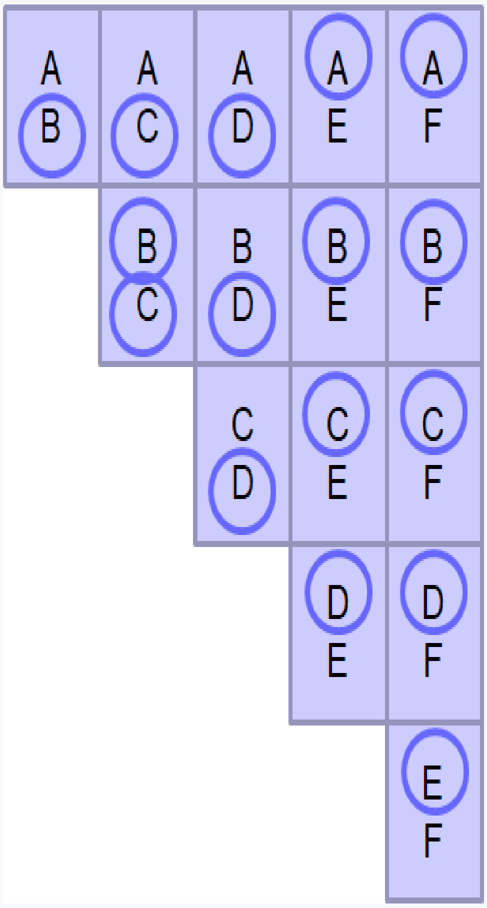
The resulting graph, constructed according to the description above:



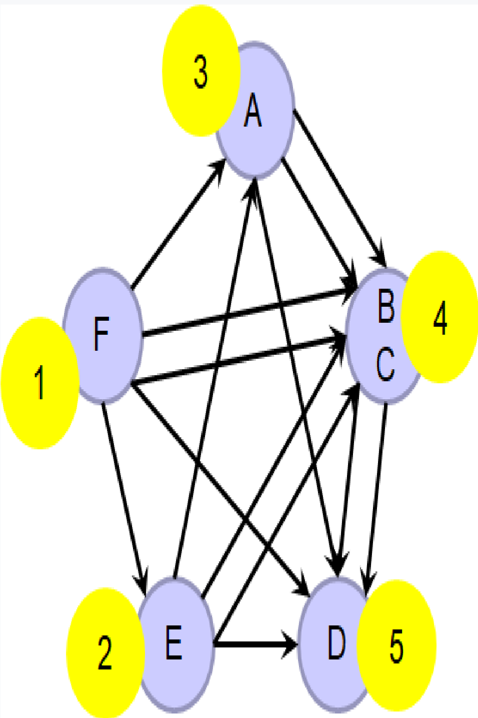
Vertices cannot be numbered, the graph is cyclic.

The only thing we can do is discard this expert's rating as unusable.

The next example presents what we will do when two criteria are evaluated by the same significance.



When building the graph, we will merge vertices with the same rating: Parallel edges must be oriented in the same way (they must be multiple):



It is evident, that in this example we can enumerate the vertices, and this criteria evaluation is correct.

The number of points for every criterion will be filled into the table:

		criteria		
expert		1	...	j
	1			
	...			
	i			γ_{ij}
	$\sum_i \gamma_{ij}$			

And the significance of every criterion will be calculated:

$$B_j = \frac{\sum_i \gamma_{ij}}{i}$$

5.2.5

Saaty's method

Saaty's method (Quantitative pairwise comparison) is also based on the evaluation of pairs of criteria. All pairs of criteria are compared and the evaluation is stored in the Saaty matrix **S** = (s_{ij}) according to the following system:

$$(s_{ij}) = \begin{cases} 1 & \text{- } i \text{ and } j \text{ are equivalent} \\ 3 & \text{- } i \text{ is weakly preferred over } j \\ 5 & \text{- } i \text{ is strongly preferred over } j \\ 7 & \text{- } i \text{ is very strongly preferred over } j \\ 9 & \text{- } i \text{ is absolutely preferred over } j \end{cases}$$

Values 2, 4, 6, and 8 are left for intermediate grades.

It is obvious that *sii* = 1 since the criterion is equal to itself.

It must hold that $s_{ji} = 1/s_{ij}$ for all *i, j*.

The value *sij* represents the approximate ratio of the weights of criteria *i* and *j*, in mathematical notation:

$$s_{ij} \approx \frac{v_i}{v_j}$$

Based on this evaluation we will use Saaty's method procedure:

First, we fill in the Saaty matrix:

1. There will be ones on the diagonal (s_{ii} = 1).
2. $s_{ij} \in \langle 1, 9 \rangle$, if *i* preferred over *j*.
3. $s_{ji} = \dfrac{1}{s_{ij}}$

We calculate the value s_i for each *i:*

$$s_i = \prod_{j=1}^{k} s_{ij}$$

We calculate for each *i:*

$$R_i = (s_i)^{1/k} = \sqrt[k]{s_i}$$

Next, we calculate:

$$\sum_{i=1}^{k} R_i$$

Finally, we determine the criteria weights by the expression:

$$v_i = \frac{R_i}{\sum_{i=1}^{k} R_i}$$

Because of the overdetermination of the system, the matrix S must be "satisfactorily consistent", with the variance estimation:

$$\sigma^2 = \frac{F}{d} = \frac{\sum_{i=1}^{k} \sum_{j>i} \left(\ln s_{ij} - (\ln v_i - \ln v_j)\right)^2}{\frac{(k-1)(k-2)}{2}}$$

$$\sigma^2 < 0{,}1 \text{ for } k = 3$$

$$\sigma^2 < 0{,}2 \text{ for } k = 4, 5, 6, 7$$

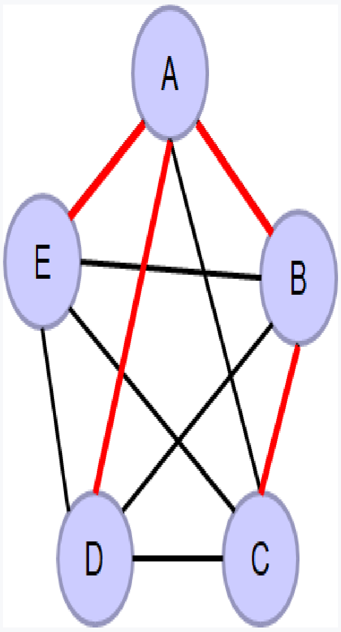
$$\sigma^2 < 0{,}3 \text{ for } k > 7$$

If we want to eliminate the overdetermination, we can use another method from Graph Theory.

It is based on the answer to a significant question: What is the minimum number of evaluations of pairs of criteria and which pairs should be compared?

The total number of comparisons represents a complete graph having $\frac{k(k-1)}{2}$ edges.

The minimum number of pairs than its spanning tree (any) having k – 1 edges:



We get an evaluation of the selected pairs, e.g.:

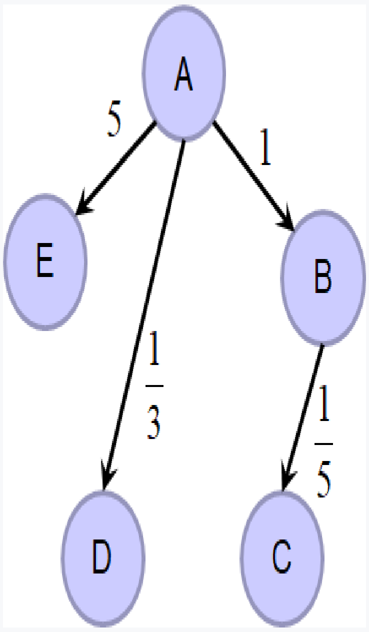
A – B = 1

B – C = 1/5

A – D = 1/3

A – E = 5

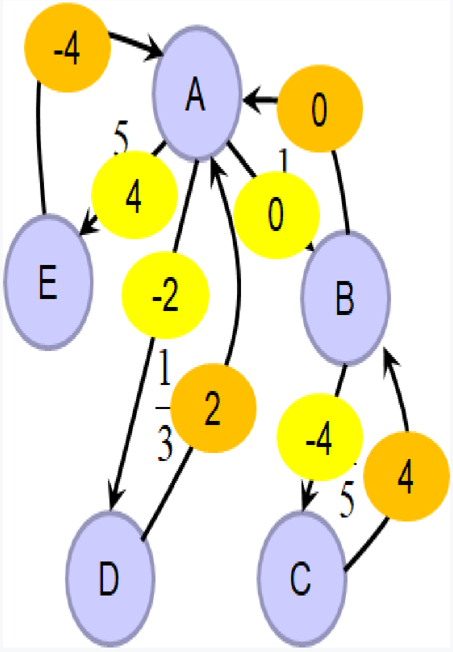
We will include the corresponding edges in the graph and mark their orientation:



We will replace the evaluation with points according to the table:

weight	points
$\frac{1}{9}$	-8
$\frac{1}{8}$	-7
...	...
$\frac{1}{2}$	-1
1	0
2	1
3	2
...	...
8	7
9	8

Next, we will add oppositely oriented edges with opposite evaluations:



We fill in Saaty's table:

We add up the points on the way from X to Y and convert them into a rating using the same table.

We will designate the lower part of the table as the evaluation of inverted values.

	A	B	C	D	E
A	1	1	1/5	1/3	5
B	1	1	1/5	1/3	5
C	5	5	1	3	9
D	3	3	1/3	1	7
E	1/5	1/5	1/9	1/7	1

Next, we proceed as standard:

Saaty matrix								weight
	A	B	C	D	E	s_i	R_i	v_i
A	1	1	0,2	0,333333	5	0,333333	0,8027	0,10917
B	1	1	0,2	0,333333	5	0,333333	0,8027	0,10917
C	5	5	1	3	9	675	3,6801	0,50046
D	3	3	0,333333	1	7	21	1,8384	0,25001
E	0,2	0,2	0,111111	0,142857	1	0,000635	0,2294	0,03119

Note: Using this procedure, we can get a rating higher than 9, or lower than 1/9.

5.2.6

The most sofisticated method for evaluation the criteria importance is:

- Saaty's method.
- Satyr's method.
- Sander's method.

5.3 Evaluating the alternatives

5.3.1

The next step is the determination of the evaluation of alternatives according to the criteria

- sometimes we have a problem obtaining objective values, in this case, expert knowledge could be used,
- verification from multiple independent sources can help to achieve objective values...

We also need to specify the type of criteria

- cost-based criterion (-) – we are minimizing,
- beneficial-based criterion (+) – we are maximizing.
- the type of criterion must be taken into account when evaluating alternatives.

The last step is ranking the alternatives. They are many methods for ranking alternatives, the easiest and most robust are:

- partial order method,
- method of the base.

📖 5.3.2

Partial order method

The partial order method (weighted by criteria importance) is the easiest method for ranking the alternatives.

According to the evaluation, we will determine the order of the alternatives for every criterion.

		criteria			
		1	...	j	
alternatives		B_1		B_j	P_i
	1				
	...				
	i			h_{ij}	

According to the filled table, we determine the weighted ranking of every alternative:

$$P_i = \sum_j B_j h_{ij}$$

The last step is to determine the order of the alternatives. Obviously, the lower the weighted rank, the better.

📖 5.3.3

Method of base

for each criterion we determine the base value (minimum, maximum, 0, most often the average):

$$h_{Bj} = \frac{\sum_i h_{ij}}{n}$$.

		criteria		
		B_1		B_j
		1	...	j
alternatives	1			
	...			
	i			h_{ij}
	h_{Bj}			

Next, we recalculate the values, based on the type of criterion:

- cost-based criterion (-):

$$z_{ij} = \frac{h_{Bj}}{h_{ij}} B_j$$

- beneficial-based criterion (+):

$$z_{ij} = \frac{h_{ij}}{h_{Bj}} B_j$$

Next, we can calculate the usefulness of all alternatives:

$$S_i = \sum_j z_{ij}$$

Obviously, the higher value the alternative achieves, the better.

5.3.4

Some special ideas

A typical question is if it has sense to use more different methods for ranking alternatives and comparing the obtained results.

The answer is obvious. No. Each method has a different basis, and results may be different.

Some problems are very sensitive to changing the values of parameters or the significancy of criteria. The problem of the sensitivity of the task to the value of the coefficients of significance can be evaluated:

- try small value changes (± 10 %).

5.3.5

When using the method of the base, the best alternative will have the usefulness of all alternatives

- highest.
- lowest.
- average value.

Examples

Chapter 6

6.1 Web portal development

🕮 6.1.1

Our company wants to install a new web portal. A tender was announced, to which four companies applied. Based on the information given by these four companies we define four alternatives evaluated by four criteria:

- Price in CZK
- Realization time in months
- References, meaning the number of previous successfully realized projects
- The quality of the design, expressed in points corresponding to the percentage of fulfillment of our requirements.

	Criteria			
Alternative	Price [CZK]	Realization time [month]	References [number]	Quality [points]
1	80000	12	0	70
2	160000	12	9	80
3	180000	15	5	65
4	240000	7	12	95

To obtain the weighting of the criteria we will organize a group of four experts and ask them to order the criteria from most important (1) to least important (4)

Their opinions are fulfilled to the table, to verify the fulfillment correctness, the checksum in every row must be 10:

	Criteria				
Expert	Price	Realization time	References	Quality	Checksum
E1	2	3	4	1	10
E2	4	3	2	1	10
E3	1	3	4	2	10
E4	1	4	3	2	10
Sum	8	13	13	6	40
Bj	0,8	0,675	0,675	0,85	

To obtain the significance of every criterion we count the sum in every column and count B_j = 1- (sum in column/sum of all columns)

To evaluate all alternatives we will use the partial ordering method. So we will determine the type of criterion and order all alternatives from 1 to 4:

	Criteria			
Alternative	Price	Realization time	References	Quality
1	1	2,5	4	3
2	2	2,5	2	2
3	3	4	3	4
4	4	1	1	1
Bj	0,8	0,675	0,675	0,85

According to the Partial ordering method we will now multiply all values by criteria importance B_j and cout the sum in every row P_i:

	Criteria					
Alternative	Price	Realization time	References	Quality	Pi	Order
1	0,8	1,6875	2,7	2,55	7,7375	3
2	1,6	1,6875	1,35	1,7	6,3375	2
3	2,4	2,7	2,025	3,4	10,525	4
4	3,2	0,675	0,675	0,85	5,4	1

The lowest value indicates the best alternative.

6.2 Place for nuclear power station

🕮 6.2.1

Three possible places for new nuclear power stations were identified and evaluated according to six criteria:

1. Number of workers
2. Maximal power in MW
3. Investment in billions CZK [mld. Kč]
4. Year operating costs in millions CZK [mil. Kč]
5. Number of evacuated residents
6. Degree of operational responsibility in points

	criteria					
Alternatives	Number of workers	Power [MW]	Investment [mld. Kč]	Operating costs [mil. Kč]	Number of evacuated residents	Degree of operational reliability [points]
1	6500	4000	90	400	5500	9
2	4800	2400	50	300	4000	7
3	7500	4800	100	500	6000	8

The expert panel will be assembled by four experts. The fuller triangle method will be used for evaluating the criteria.

Each expert receives a list of all pairs of criteria and marks the more important of them (one point) or marks both as equally important (half a point each).

The number of given points is written in the next table. To check the correct number of points we will calculate the checksum in every row. Having six criteria, expert must give 15 points (5 + 4 + 3 + 2 + 1):

	criteria						
Expert	Number of workers	Power [MW]	Investment [mld. Kč]	Operating costs [mil. Kč]	Number of evacuated residents	Degree of operational reliability [points]	checksum
E1	2	2	3	2	4	2	15
E2	3,5	2	4,5	3,5	1	0,5	15
E3	1	3	1	2	3	5	15
E4	3,5	2,5	3	1	1	4	15
Bj	2,5	2,375	2,875	2,125	2,25	2,875	

The importance coefficient B_j is calculated as the sum in the column divided by the number of experts.

To evaluate the alternatives we will use the base method.

In the first table we will identify the type of criteria and count the base as the average value of every criterion:

	criteria						
Alternatives	Number of workers	Power [MW]	Investment [mld. Kč]	Operating costs [mil. Kč]	Number of evacuated residents	Degree of operational reliability [points]	checksum
1	6500	4000	90	400	5500	9	15
2	4800	2400	50	300	4000	7	15
3	7500	4800	100	500	6000	8	15
Bj	2,5	2,375	2,875	2,125	2,25	2,875	15
type	-	+	-	-	-	+	
base	6266,67	3733,333	80	400	5166,67	8	

The last step is the recounting of all values based on the type of criterion:

	Criteria							
Alternatives	Number of workers	Power [MW]	Investment [mld. Kč]	Operating costs [mil. Kč]	Number of evacuated residents	Degree of operational reliability [points]	Si	Order
1	2,41	2,54	2,56	2,125	2,11	3,23	14,98	2
2	3,26	1,53	4,6	2,83	2,91	2,52	17,65	1
3	2,09	3,05	2,3	1,7	1,94	2,88	13,95	3

Now we can count the usefulness of alternatives S_i and order alternatives. The highest value is the best.

6.3 Your own example

6.3.1

Define your own multi-criteria decision-making task. Use your classmates as experts and solve this task. Were the multi-criteria methods helpful?

Literature

Chapter 7

7.1 Literature sources

🕮 7.1.1

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